Paulson, Morris James
An investigation of the standard of achievement in the lower limit of the B group in Grade IX Mathematics in June 1948.

UNIVERSITY OF ALBERTA

AN INVESTIGATION OF THE STANDARD OF ACHIEVEMENT AT THE LOWER LIMIT OF THE B GROUP IN GRADE IX MATHEMATICS IN JUNE, 1948.

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF EDUCATION

FACULTY OF EDUCATION

BY

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UNIVERSITY OF ALBERTA

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The undersigned hereby certify that they have read and do recommend to the Committee on Graduate Studies for acceptance, a dissertation " An Investigation of the Standard of Achievement at the Lower Limit of the B Group in Grade IX Mathematics in June, 1948" submitted by Morris James Paulson, B.Ed., in partial fulfilment of the requirements for the degree of Master of Education.

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THE PURPOSE AND THE PROCEDURE OF THE INVESTIGATION

I. The Purpose of the Investigation

The purpose of this investigation is to determine by an analysis of the lowest 300 B group examination papers, the existing standards of mathematical achievement as found in the Grade IX Mathematics papers for June,1948. This investigation will be concerned with those 157 candidates having a raw score of 40, and 143 of the candidates having a raw score of 41. These raw scores correspond to transmuted or percentage scores of 45 and 46 respectively.

The problems which require solution in regards to this investigation are, namely:

a. What do the students at this level know?

b. What do the students at this level not know?

c. Are definitions meaningful to the students?

d. Do the students know proofs and constructions?

e. Does the students' work show any evidence of logical thought, or does their work reflect only memorization?

f. Does the examination paper test the students' powers to specialize and apply abstract theory to particular problems and situations?

g. Does the examination paper test the students' powers to generalize or theorize?

h. Was the examination fair?

To answer these questions the prescribed text-book, the examination booklet, and the 300 students'

CHAPTER I

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f. Does the examination paper test the students' powers to specialize and apply abstract theory to particular problems and situations?

g. Does the examination paper test the students' powers to generalize or theorize?

h. Was the examination fair?

To answer these questions the prescribed text book, the examination booklet, and the 300 students'

answer papers were critically analyzed under the following main headings:

a. Relationships

b. Concepts or technical vocabulary

c. Routines or techniques.

II. The Examination Paper

The Grade IX mathematics paper for 1948 consists of two sections, Section A and Section B. Section A contains 40 multiple choice questions worth 2 marks each and arranged for machine scoring. Section B contains 13 problems with a total raw score of 39 marks. These 13 questions are not machine scored, and a suitable place is provided for rough work and the correct answer. In all, the paper consists of 53 questions, totalling 119 marks.

III. Selection of the Data

Nine thousand six hundred and forty four (9644) students, using the prescribed text " Mathematics for Today" by Lazerte and Betz, wrote the Grade IX mathematics examination in Alberta schools in 1948. The marking, ranking, and division of scores into the various grades were done by the Department of Education prior to the commencement of this investigation.

In order to select the raw data (the 300 examination papers), it was necessary to obtain the individual score cards for all the Grade IX students, from the Dept. of Education. Examination of these cards showed that a raw score of 40 comprised the lower limit of the grade B mathematics students. All the score cards having a raw score of 40 (157), and 143 of the cards having raw scores of 41 were chosen to make up the 300 lowest grade B

raw scores. After obtaining the file number of each of these 300 score cards, it was then possible to search out the individual examination papers. These 300 papers comprised the raw data for this investigation. The distribution of these 300 scores is shown in Table I below.

TABLE I

DISTRIBUTION OF SCORES

Raw Score	Transmuted Score	Number of Papers
40	45	157
41	46	143
		Total 300

IV. Range of Raw Scores

The highest possible raw score was 119, and the actual range of raw scores was from a low of 10 to a high of 102. These two scores correspond to transmuted or percentage scores of 5 and 100 respectively.

V. Distribution of Transmuted Scores

The formation of the transmutation table was based on the distribution of raw scores as found in a random selection of 1000 papers taken from a variety of centers throughout the province, namely: villages and small towns, larger towns, Calgary, Edmonton, and various rural areas. A distribution table and a cumulative frequency table, based on the results of the 1000 mentioned papers were made. The Examinations Branch had previously determined what perdentage of students would be given H, A, B, C, D, and knowing these percentages, it was possible to divide the cumulative frequency table into the five separate standings. The Board had also decided what range of transmuted scores would be included in each of the five above groups.

raw scores. After obtaining the file number of each of these 300 score cards, it was then possible to secure all the individual examination papers. These 300 papers comprised the raw data for this investigation. The distribution of these two groups is shown in Table 1 below.

TABLE 1

DISTRIBUTION OF GROUPS

Test Group	[illegible]	Number of Papers
10	[illegible]	157
11	[illegible]	143
		Total 300

(1). Range of the Scores

The highest possible raw score was 150, and the actual range of the scores was from a low of [illegible] to a high of [illegible]. These raw scores were converted to [illegible] scores of [illegible] and 100 respectively.

2. Distribution of [illegible]

The formation of the [illegible] table was based on the distribution of the scores as found in a random selection of 1000 scores taken from a variety of schools throughout the province, cities, villages and small towns, county towns, [illegible], and [illegible]. A distribution table and a cumulative frequency curve, based on the results of the 1000 [illegible] were made. The [illegible] had previously [illegible] that percentages of students would be given A, B, C, D, and [illegible] these percentages, it was possible to divide the cumulative frequency table into the five [illegible] classifications. The board had also [illegible] what range of [illegible] scores would be included in each of the five grade groups.

A straight line graph was next constructed with the ordinate representing the range of raw scores from 0 to 100, and the abscissa representing the range of transmuted scores from 0 to 100. With this graph it was then possible to find the transmuted score corresponding to any given raw score, and from this obtain a distribution of transmuted scores for the province as a whole.

VI. Ranking and Range of Scores

TABLE II

RANKING AND RANGE OF SCORES

% Score	Raw Score	Grade	Nol of Papers	% of Total
90-100	75-102	H	959	9.94
60-89	57-74	A	2484	25.76
45-59	40-56	B	3194	33.12
30-44	28-39	C	1940	20.12
0-29	10-27	D	1067	11.06
			Total 9644	100.00

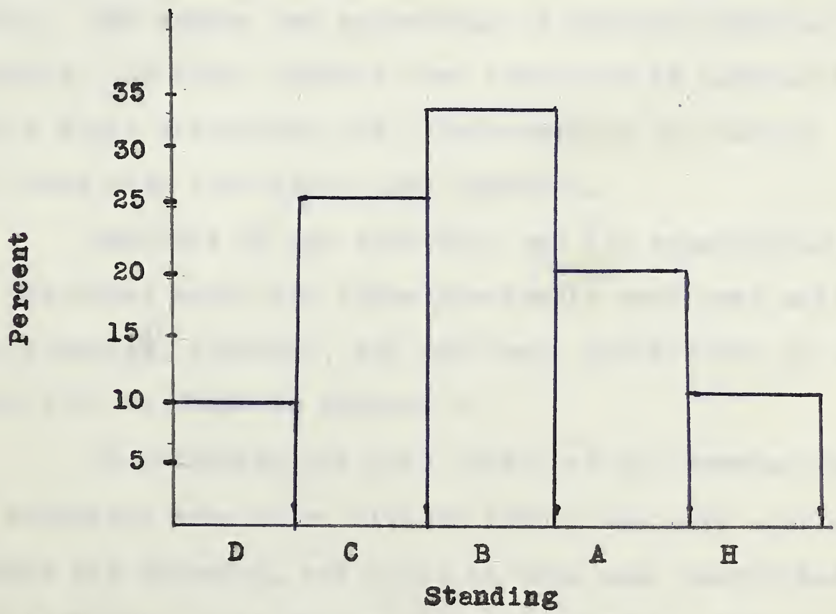


Figure 1. Distribution of standings or grades.

VII. Procedure

Analysis of the raw data in this investigation was performed in four different steps:

a. Analysis of the distribution of scores on the individual test items

b. Analysis of the individual answers on each test item

c. Analysis of the text-book and examination paper for relationships, concepts and routines

d. Analysis of the main topics on the examination paper.

Analysis of the distribution of scores on each individual test item was developed by using tables and graphs to show the frequency of zero scores, partial scores, and perfect scores on each of the 53 examination questions.

Analysis of the individual answers on each test item was done question by question for each of the 300 papers. The number and percentage of correct answers, no attempts, and other answers were expressed in tabular form, with a short discussion and interpretation of results following each individual test question.

Analysis of the text-book and the examination paper was performed under the three previously mentioned main heading relationships, concepts, and routines, definitions of each which will be found in Chapter V.

In analysing the main topics of the examination, the questions were first divided into two main sections, Algebra and Geometry, and these in turn were subdivided into individual main topics for sub-analysis. Chapter VIII explains fully how the "mean total score" for the

300 students on each individual topic was calculated and expressed as a percentage of the total raw score possible for that particular topic.

CHAPTER II

A REVIEW OF PREVIOUS STUDIES

A short review of several previously written theses on the related topic of text-book and examination paper analysis will "pin-point" the conclusions reached in former studies. Although some of these theses do not deal directly with the raw data of the grade IX mathematics examinations, they all have conclusions which bear directly on the work of the grade IX mathematics teacher.

1. A Determination of Generalizations Basic to the Mathematics Curricula of the Intermediate and Senior High Schools of Canada: Part 3- Generalizations of Grade IX, by Gordon French- 1944.

Mr. French made an analysis of all the grade IX mathematics texts used by the nine provinces of Canada, and he divided the obtained subject matter into three main groups: generalizations, language, and information. The writer found some difficulty in defining each of these terms, and in his introduction Mr. French states, "At the beginning of the study an attempt was made to define each of these three terms for the purpose of distinguishing them. However as the study progressed it became apparent that the simplest solution was that of classifying the concepts arbitrarily either as generalizations, language or information owing to the difficulty of border-line cases. For example, such terms as parallel lines, angle, perpendicular, were classified as language although each is as rich in ideas as many of the generalizations".

455 generalizations under 45 main headings were tabulated as those basic to the grade IX mathematics curricula of

Intermediate and High Schools of Canada, and 547 terms were classified as language instead of generalizations.

2. A Determination of Generalizations Basic to the Mathematics Curricula of the Intermediate and Senior High Schools of Canada: Part 4- Generalizations of Grade X, by Ottar Massing-1945.

Mr. Massing conducted a survey similar to that of Mr. French, only his subject matter was concerned with the grade X mathematics course in the various provinces of Canada. He also classified the subject matter into generalizations and information and language. A total of 478 generalizations under 43 main headings were tabulated as basic to the grade X mathematics courses of Canada, and 608 language terms were classified, 198 being new terms and 410 listed as terms which were found in earlier mathematics courses.

3. A Report on Algebra 3 Papers of June 1938, by S.W. Hooper-1941.

Although Algebra 3 bears little relationship to the grade IX mathematics, Mr. Hooper's conclusions are well worth repeating. He states, "If the paper is considered a fair test of the principles involved in the course, certainly these results indicate a very serious ignorance of the principles underlying the course and a decided weakness in mathematical manipulation".

4. Graphs in General Mathematics, by H.E. Miller-1941.

Mr. Miller made a survey of the abilities of various grades to understand and interpret graphical concepts. He gave three groups of tests over a period of time to various

groups of students from grade 7 to grade 12, and from the results he concludes that the problem of graphs has not been well handled in the Alberta school system, and also that the idea of negative numbers though very confusing to a great number of students, could much better be taught if it were related more closely to the concept of graphs. Miller's concluding remarks were, "I believe that the results of the tests prove that the graphical concept is not well enough understood even by senior high school pupils. I believe that the topic should be treated more fully at one point in the high school course, possibly at the grade IX level."

5. Student Attainment in the Algebra 2 Examination of 1944, by Velma Miller-1946.

One of Miss Miller's concluding statements bears directly on this thesis. She concludes,"This survey indicates a weak mathematical background for most students. Perhaps teachers of mathematics might take time to stress more of the fundamentals.---- The finding of so many mechanical errors would also indicate a need for insisting upon accuracy of details and for more drill upon fundamentals during earlier school years".

6. Summaries of Investigations in the Mathematical Field, submitted by Dr. M.E. Lazerte to the Carnegie Corporation- May, 1936.

A review of this thesis, with regards to the present investigation may best be presented by separate quotations from Dr. Lazerte's summary. On page 5 Dr. Lazerte states," It would appear that pupils are not being trained in independent thinking. Faulty analysis, or a

groups of students from grade 1 to grade 12, and from the results he concludes that the problem of errors has not been well handled in the Alberta school system, and also that the type of negative numbers found when dealing with a great number of students, would seem better to conduct it with relation more closely to the comments of [illegible]. Wilson's concluding remarks were, "I believe that the results of the tests prove that the graphical concepts is not well enough understood even by senior high school pupils. I believe that the topic should be treated more fully at some point in the high school course, possibly at the grade IX level."

5. Student attainment in the [illegible] of 1951, by Glen Wilson-1952.

One of Glen Wilson's conclusions [illegible] directly on this thesis. He concludes, "This survey indicates a weak mathematical background for most students. Perhaps teachers of mathematics cannot look with too great [illegible] of the fundamentals.--- The findings of so many successive surveys would also indicate a need for insistence upon attainment of skills and the more drill upon fundamentals during earlier school years."

6. [illegible] 1951, submitted by Dr. [illegible] Department [illegible] 1952.

A review of this [illegible], also reported in the [illegible] investigation [illegible] by [illegible] from Dr. [illegible]'s summary. On page 2 he states, "It would appear that pupils are not being [illegible] in fundamental [illegible]. [illegible]

complete absence of analysis, accounts for the largest single group of errors made. Pupils are not learning to grasp the relationship between the various factors entering into a given problem; in a great many instances they see no problem at all". On page 6 the report continues, " Pupils are very weak in the matter of ratios".

On page 99, one reads, "This ability (to see relationships) which might perhaps be more accurately described as a composite of related abilities, is fundamental to mathematical thinking". On page 103 the writer continues, "The most difficult phase (of problem solving) is without a doubt the analysis of the problem with a view to discovering the relationships involved, and the statement of this relationship in an equation".

On page 107 Dr. Lazerte concludes his investigation with the following statement, "This study has consistently indicated a great need for two changes:

a. a transference of emphasis to functional thinking in mathematics

b. more attention to graphical work through-out all phases of mathematics".

CHAPTER III

ANALYSIS OF THE DISTRIBUTION OF SCORES ON THE INDIVIDUAL TEST ITEMS

The following frequency tables and figures for each test item on the examination paper show the distribution of marks for the 53 questions on the examination. Test items 1-40 inclusive received either full credit of 2 marks each, or no marks at all. Test items 41-53 inclusive varied in value from one to six marks, with credit given for partially correct work.

TABLE III

FREQUENCY OF SCORES ON QUESTIONS 1-8.

Question	1	2	3	4	5	6	7	8
Score 0	136	88	39	278	116	244	171	67
2	164	212	261	22	184	56	129	233

TABLE IV

FREQUENCY OF SCORES ON QUESTIONS 9-16.

Question	9	10	11	12	13	14	15	16
Score 0	209	36	178	68	58	201	249	205
2	91	264	122	232	242	99	51	95

TABLE V

FREQUENCY OF SCORES ON QUESTIONS 17 -24

Question	17	18	19	20	21	22	23	24
Score 0	127	179	164	216	229	188	162	193
2	173	121	136	84	71	112	138	107

TABLE VI

FREQUENCY OF SCORES ON QUESTIONS 25-32

Question	25	26	27	28	29	30	31	32
Score 0	195	287	208	105	219	264	175	146
2	105	13	92	195	81	36	125	154

TABLE VII

FREQUENCY OF SCORES ON QUESTIONS 33-40

Question	33	34	35	36	37	38	39	40
Score 0	260	129	183	259	180	242	193	140
2	40	171	117	41	120	58	107	160

TABLE VIII

FREQUENCY OF SCORES ON QUESTIONS 41-44

Question	41	42	43	44
Score 0	173	204	114	74
1	127	96	186	226

TABLE IX

FREQUENCY OF SCORES ON QUESTION 45

Question	45
Score 0	232
1	1
2	67

TABLE X

FREQUENCY OF SCORES ON QUESTIONS 46a,b.

Question	46a	46b
Score		
0	242	256
1	58	44

TABLE XI

FREQUENCY OF SCORES ON QUESTION 47

Question	47
Score	
0	180
1	37
2	0
3	83

TABLE XII

FREQUENCY OF SCORES ON QUESTIONS 48-49

Question	48	49
Score		
0	224	190
1	0	0
2	76	110

TABLE XIII

FREQUENCY OF SCORES ON QUESTION 50

Question	50
Score	
0	230
1	11
2	4
3	0
4	55

TABLE XIV

FREQUENCY OF SCORES ON QUESTIONS 51-52

Question	51	52
Score		
0	235	260
1	52	39
2	5	1
3	2	0
4	2	0
5	3	0
6	1	0

TABLE XV

FREQUENCY OF SCORES ON QUESTION 53a,b,c,d.

Question '	53a	b	c	d
Score				
0	293	260	279	297
1	0	0	0	0
2	7	40	21	3

TABLE XVI

FREQUENCY OF TOTAL SCORES ON QUESTION 53

Question	53
Score	
0	234
2	60
4	6
6	0
8	0

The following frequency tables and graphs show the distribution of perfect scores, zero scores, and no attempts for each of the test questions 1-53 inclusive.

TABLE XVII

FREQUENCIES OF PERFECT SCORES- QUESTIONS 1-53

Question	N	Question	N
1	164	30	36
2	212	31	125
3	261	32	154
4	22	33	40
5	184	34	171
6	56	35	117
7	129	36	41
8	233	37	120
9	91	38	58
10	264	39	107
11	122	40	106
12	232	41	127
13	242	42	96
14	99	43	186
15	51	44	226
16	95	45	67
17	173	46a	58
18	121	46b	44
19	136	47	83
20	84	48	76
21	71	49	110
22	112	50	55
23	138	51	1
24	107	52	0
25	105	53a	7
26	13	53b	40
27	92	53c	21
28	195	53d	3
29	81		

"N" represents the total number of candidates making perfect scores on the particular questions.

Percentage of Students

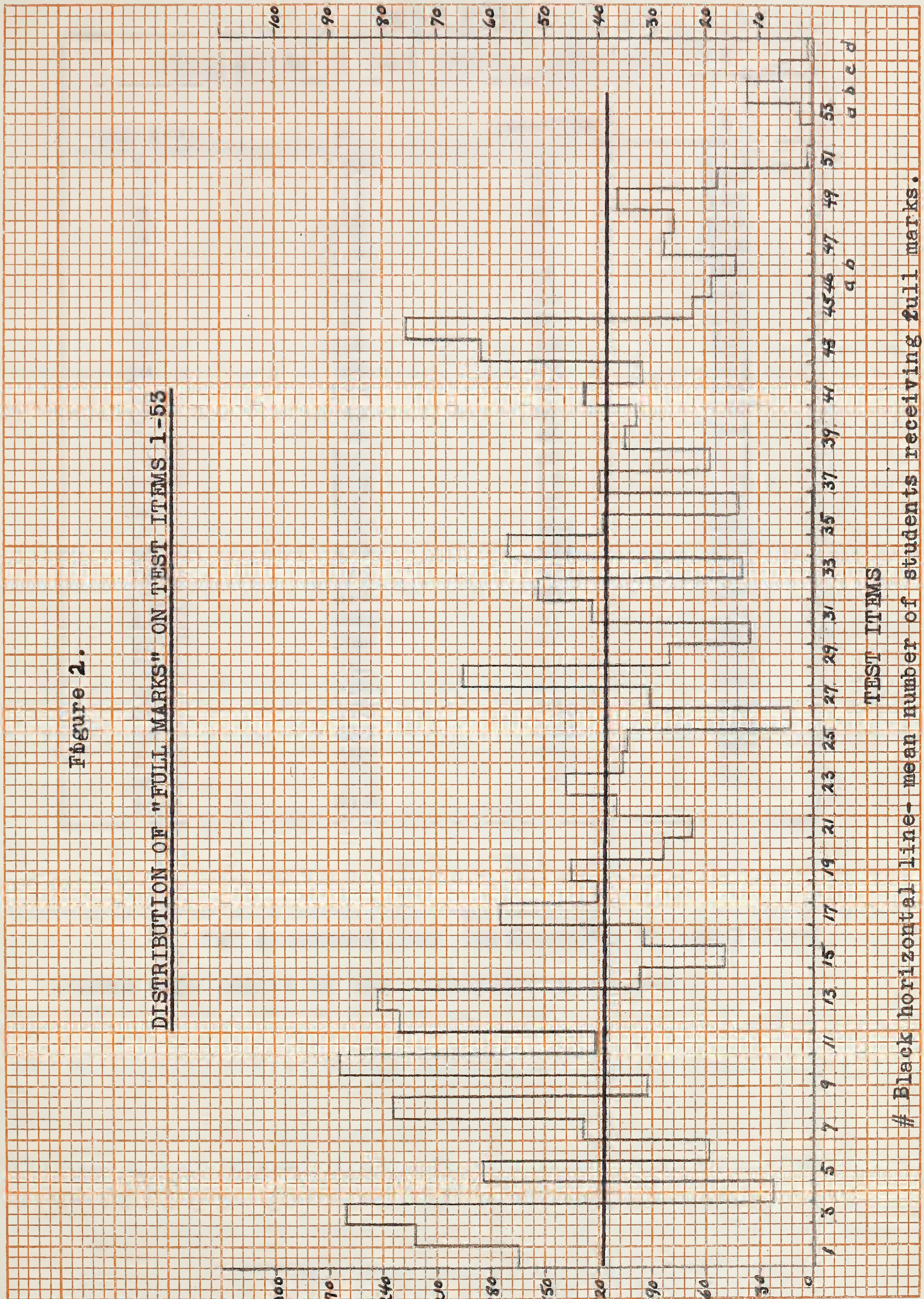


Figure 2.

DISTRIBUTION OF "FULL MARKS" ON TEST ITEMS 1-53

Black horizontal line- mean number of students receiving full marks.

Number of Students

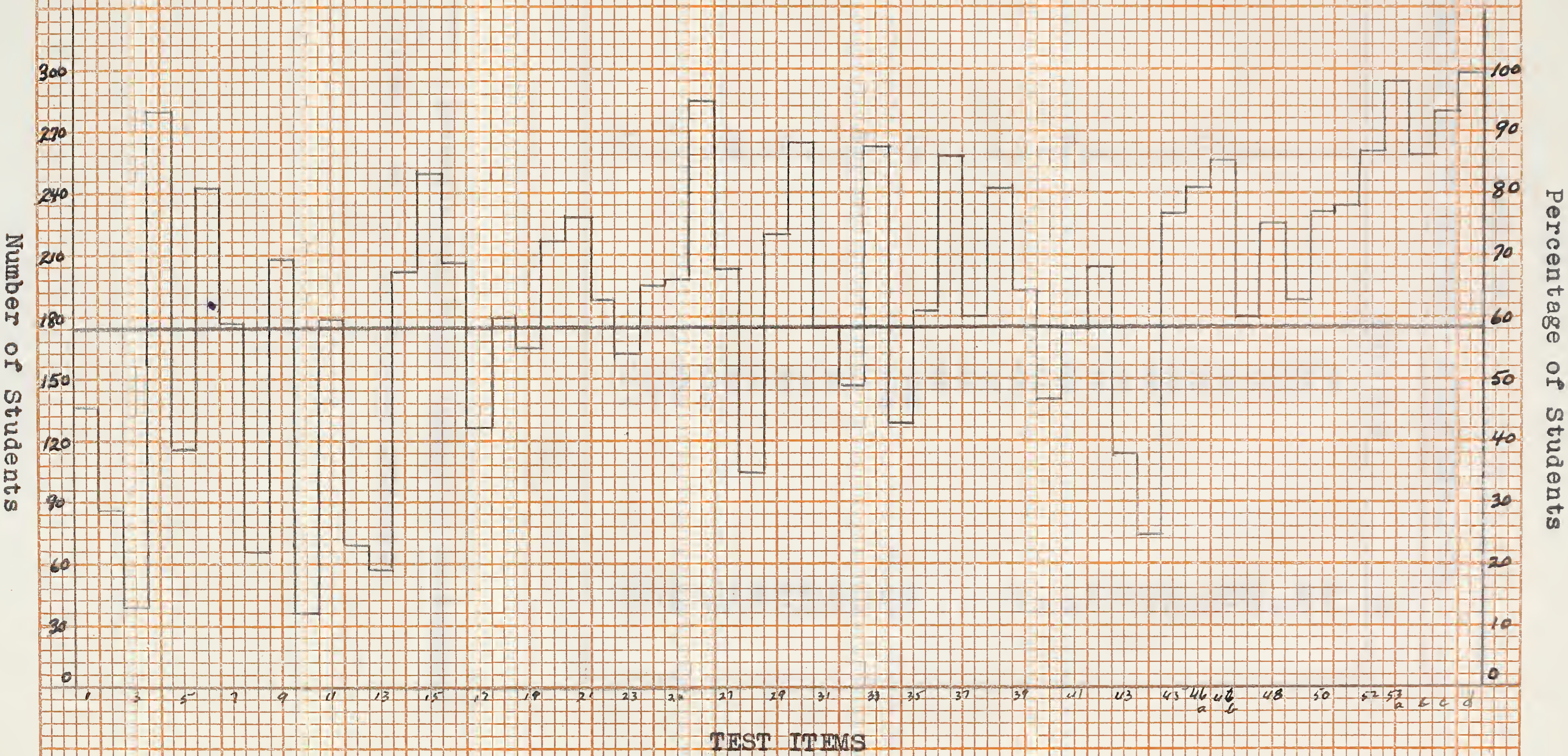
TABLE XVIII

FREQUENCIES OF ZERO SCORES- QUESTIONS 1-53			
Question	N	Question	N
1	136	30	264
2	88	31	175
3	39	32	146
4	278	33	260
5	116	34	129
6	244	35	183
7	171	36	259
8	67	37	180
9	209	38	242
10	36	39	193
11	178	40	140
12	68	41	173
13	58	42	204
14	201	43	114
15	249	44	74
16	205	45	232
17	127	46a	242
18	179	46b	256
19	164	47	180
20	216	48	224
21	229	49	190
22	188	50	230
23	162	51	235
24	193	52	260
25	195	53a	293
26	287	53b	260
27	208	53c	279
28	105	53d	297
29	219		

"N" represents the total number of candidates making zero scores on the particular questions.

Figure 3.

DISTRIBUTION OF "ZERO SCORES" ON TEST ITEMS 1-53



#- Black horizongal line- mean number of students receiving zero scores.

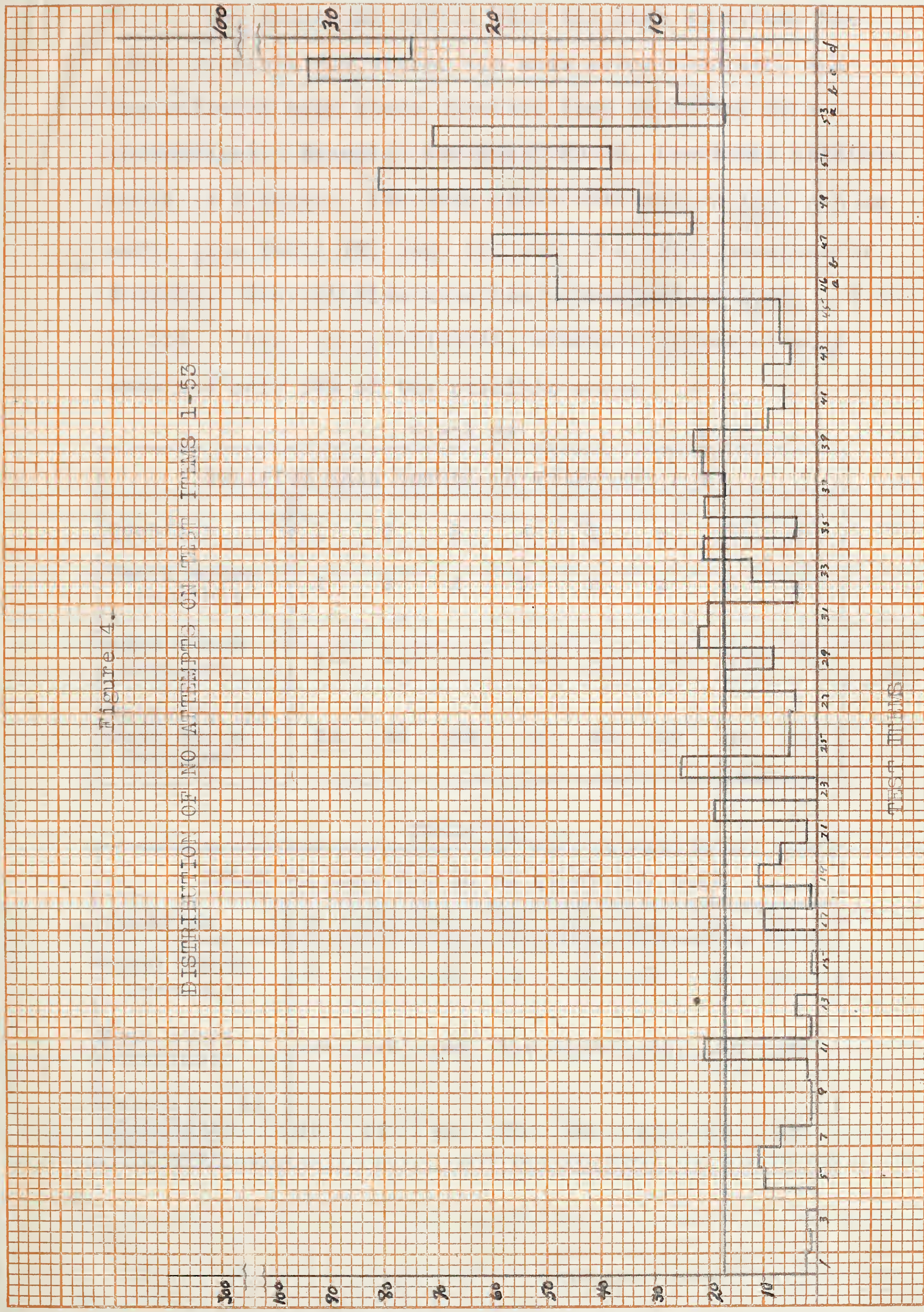
TABLE XIX

FREQUENCIES OF NO ATTEMPTS- QUESTIONS 1-53

Question	N	Question	N
1	2	30	22
2	1	31	20
3	2	32	4
4	0	33	12
5	10	34	21
6	11	35	4
7	7	36	21
8	1	37	17
9	1	38	21
10	2	39	23
11	21	40	9
12	1	41	6
13	4	42	10
14	0	43	5
15	1	44	7
16	0	45	7
17	10	46a	48
18	1	46b	48
19	11	47	60
20	7	48	23
21	2	49	33
22	19	50	81
23	0	51	38
24	25	52	71
25	5	53a	17
26	5	53b	26
27	4	53c	94
28	17	53d	75
29	8		

"N" represents the total number of candidates making no attempts on the particular questions.

Percentage of Students



Number of Students

Figure 4.

DISTRIBUTION OF NO ATTEMPTS ON TEST ITEMS 1-53

TEST ITEMS

Black horizontal line- mean number of students making no attempts

The following tables show for each question, the total possible raw score, the mean score obtained, and the relative mean score obtained (and expressed as a percentage). Example: For question 45, the total possible score is 2 marks. The mean score obtained is $\frac{135}{300} = 0.45$, marks (see table IX, page 12) out of a total possible of 2 marks. The relative mean score obtained is $\frac{0.45}{2} = 0.23$ or 23%. Thus, on question 45, students on the average obtained only 23% of the possible score.

TABLE XX

RELATIVE MEAN SCORES ON QUESTIONS 1-8

Question	1	2	3	4	5	6	7	8
Total Score Possible	2	2	2	2	2	2	2	2
Mean Score Obtained	1.1	1.4	1.7	.14	1.2	.38	.86	1.6
Relative (%) Mean Score Obtained	55	70	85	7	60	19	43	80

TABLE XXI

RELATIVE MEAN SCORES ON QUESTIONS 9-16

Question	9	10	11	12	13	14	15	16
Total Score Possible	2	2	2	2	2	2	2	2
Mean Score Obtained	.6	1.8	.82	1.5	1.6	.66	.34	.64
Relative (%) Mean Score Obtained	30	90	41	75	80	33	17	32

TABLE XXII

RELATIVE MEAN SCORES ON QUESTIONS 17-24

Question	17	18	19	20	21	22	23	24
Total Score Possible	2	2	2	2	2	2	2	2
Mean Score Obtained	1.2	.80	.90	.56	.48	.74	.92	.72
Relative (%) Mean Score Obtained	6	40	45	28	24	37	46	36

TABLE XXIII

RELATIVE MEAN SCORES ON QUESTIONS 25-32

Question	25	26	27	28	29	30	31	32
Total Score Possible	2	2	2	2	2	2	2	2
Mean Score Obtained	.70	.08	.62	1.3	.54	.24	.84	1.0
Relative (%) Mean Score Obtained	35	4	31	65	27	12	42	50

TABLE XXIV

RELATIVE MEAN SCORES ON QUESTIONS 33-40

Question	33	34	35	36	37	38	39	40
Total Score Possible	2	2	2	2	2	2	2	2
Mean Score Obtained	.26	1.1	.78	.27	.80	.38	.71	.53
Relative (%) Mean Score Obtained	13	55	39	14	40	19	36	27

TABLE XXV

RELATIVE MEAN SCORES ON QUESTIONS 41-48

Question	41	42	43	44	45	46	47	48
Total Score Possible	1	1	1	1	2	2	3	2
Mean Score Obtained	.42	.32	.62	.75	.45	.34	.95	.50
Relative (%) Mean Score Obtained	42	32	62	75	23	17	32	25

TABLE XXVI

RELATIVE MEAN SCORES ON QUESTIONS 49-53

Question	49	50	51	52	53
Total Score Possible	2	4	6	6	8
Mean Score Obtained	.73	.80	.32	.14	.48

CHAPTER IV

ANALYSIS OF THE INDIVIDUAL ANSWERS ON EACH TEST ITEM OF THE EXAMINATION PAPER

As mentioned previously, the examination paper was divided into two sections, sections A and B. Section A of the paper contained questions 1-40 inclusive, and answers to these questions were recorded on separate answer sheets provided. Each of these first forty questions had 5 suggested answers, only one of which was supposedly correct. The correct answer in each case was to be selected and recorded in the proper manner on the answer sheet, as shown in the sample on the front of the examination booklet.

Section B of the paper contained questions 41-53 inclusive, and complete solutions for each of these questions were to be shown in the spaces provided in the booklet. The method of analysis was straight forward. One question at a time was considered. Each answer paper was studied carefully, and the various solutions recorded. The total number of students making each type of error was found, and this number was also listed as a percentage of the 300 students concerned with this investigation.

As well as listing the various errors found in the answers, the number and percent of students that had the correct answer, and that had made no attempt at answering the question were also recorded. Where only a few papers showed the same type of error, these were classified as miscellaneous errors. This procedure was followed for each of the 53 questions.

One difficulty arises in such an analysis of answers- that is the problem of guessing. With 300 students

and a multiple choice of 5 answers for each question in section A, the law of averages says that by chance alone, there could be 60 students with the correct answer, and 60 students in each of the other four divisions of suggested answers. If it so happened that we found a problem answered such that the results were approximately evenly distributed as such, would we justified in saying definitely that at least 60 students knew how to solve the problem correctly? This very important factor must be kept in mind for the first 40 questions on the examination paper. We must not be too definite in our statements, regarding the standard of achievement, when results are based solely on multiple choice type questions, as in section A. Section B must also play a vital part in determining the standard of adhievement of these 300 students.

<u>Question 1:</u> The product of 98.05 and 12.44 is....

TABLE XXVII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	2	1
Chose answer a. 12.19742	4	1
Chose answer b. 121.9742	60	20
Chose answer c. 1219.742 (correct)	164	55
Chose answer d. 12197.42	6	2
Chose answer e. 12197.420	64	21

The form of this question will be criticized in a later chapter, but the results themselves are disappointing. Considering that this question is actually a review of grade 7 and 8 mathematics, it is surprising that only 55% of these B standing students chose the correct answer. The law of averages states that chance alone can be responsible for approximately 60 students "placing" in each of the 5 categories of multiple choice. We may hazard a guess and say that those 120 students choosing the suggested answers "b" and "e" had no more idea of the correct answer than those students who chose the answers "a" and "d".

This poor showing in the handling of decimals may be attributed to a lack of training in the basic fundamentals through-out public school, and especially to the laxity of many teachers in not stressing "rough approximations" as a check on the pupil's mechanics.

Question 2: The complement of an angle of 40 degrees is an angle of .... degrees.

TABLE XXVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	1	0
Chose answer a. 140 degrees	53	18
Chose answer b. 90 degrees	16	5
Chose answer c. 60 degrees	13	4
Chose answer d. 320 degrees	5	2
Chose answer e. 50 degrees (correct)	212	71

Question 2 was quite well done. 71% of these 300 students obtained the correct answer, and only 1 person made no attempt to solve the problem. This one person evidently did not understand the concept of "complemement" and the 18% that chose the answer as "a" confused the meaning of complement with supplement, and naturally marked the answer as a.(140 degrees.)

Question 3: 1/2 + 1/3 + 1/5 =

TABLE XXIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	2	1
Chose answer a. 3/10	12	4
Chose answer b. 1 1/30 (correct)	261	87
Chose answer c. 1 4/10	12	4
Chose answer d. 1/30	9	3
Chose answer e. 1/10	4	1

Question 3 was a review of grade 7 and 8 mathematics, and this time the students made a much better showing on their basic fundementals with 87% of our 300 students choosing the correct answer. The incorrect answers can be attributed to guessing, rather than errors in calculation as we see that the 37 students who did attempt to answer the question, though incorrectly, are fairly well distributed among the four remaining suggested answers.

Question 4: Which of the following statements is true?

TABLE XXX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	0	0
Chose answer a. The area of the walls of a room is the length of the room multiplied by the width of the room.	111	37
Chose answer b. The area of the four walls is the product of the perimeter of the room and its height. (correct)	22	7
Chose answer c. The area of the four walls is the product of the length, width, and height.	65	22
Chose answer d. The area of the four walls is four times the area of any one wall.	63	21
Chose answer e. The area of the four walls of a room is the sum of its length and width multiplied by its height.	39	13

Question 4 was the second most poorly done of the 40 multiple choice questions. Only 7% chose the correct answer. This was a question designed to test the student's ability to specialize and apply a somewhat abstract theory to a concrete problem. The results prove that these 278 students with incorrect answers, are not able to apply theory learned in the classroom to practical out-of-school situations.

Poor reading habits and inability to comprehend verbal problems may have been a deciding factor with many of the students.

Question 5: When 16 is multiplied by N the product is 96. What is the product when 16 is multiplied by $(N \neq 2)$?

TABLE XXXI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPT, & ERRORS

	N	%
No Attempts	10	3
Chose answer a. 112	46	15
Chose answer b. 128 (correct)	184	62
Chose answer c. 144	10	3
Chose answer d. 32	45	15
Chose answer e. 160	5	2

Although this problem contains only one step beyond the "solution of easy equations" as found in the first part of the text-book, only 62% of the 300 students were able to choose the correct answer. Had the problem asked to find "N", it is likely that a great number of students would have chosen the correct answer, but when these same students were asked to apply a little logical thought to a problem instead of memory learning, they were at a loss as to the correct procedure. This is borne out by the fact that 10 persons made no attempt at all to find a solution.

If the 46 persons choosing answer "a" got the answer "112" by blindly adding the only two numbers involved in the equation, we have further evidence that many students were completely lost as to the meaning of the question. This would also apply to those 45 students who chose answer "d" and presumably got the answer "32" by multiplying 16 by 2, but without any logical justification for performing this operation.

Question 6: A circle has a diameter of 15 inches. Its area (sq. in.) is....

TABLE XXXXII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	11	4
Chose answer a. 7.5π	27	9
Chose answer b. 15π	87	29
Chose answer c. $(7.5)^2\pi$ (correct)	56	19
Chose answer d. $(15)^2\pi$	74	25
Chose answer e. $(7.5\ )^2\pi$	45	15

It is surprising that only 19% of the 300 B group students understood the basic relationship between the diameter of a circle and its area, or the radius of a circle and its area. There were 87 students, almost 30% of our group, that confused the formula for the area of a circle with the circumference of a circle and thus chose suggested answer "a"; while almost 25% presumably knew the correct formula for finding the area of a circle, but carelessly chose the radius as 15 instead of 7.5. Those choosing answer "e" presumably had an idea of the formula to be used but were either careless regarding the position of the parentheses, or else they did not understand the value of "pi" within the parentheses.

Question 7: Which of the following ratios is equal to 5/8?

TABLE XXXIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	7	2
Chose answer a. 3/8	8	3
Chose answer b. 4/7	95	32
Chose answer c. 1/3	12	4
Chose answer d. 1.5/2.4 (correct)	129	43
Chose answer e. 8/5	49	16

Question 7 was poorly done. 43% of the 300 students found the right answer, and much of this percentage may be attributed to lucky guessing on the part of the 129 students concerned. The surprising answers are "b" and "e". We may presume that the 95 students who chose answer "b" were somewhat confused with the axiom, "if equals be subtracted from equals, then the remainders are equal", and accordingly subtracted an equal quantity "1" from both numerator and denominator and got the result 4/7. Those 49 students choosing the answer 8/5 likely thought that inverting a fraction does not change its value. In these last two examples we see again, ample justification for believing that very few of these 300 students use any logical thought in solving their problems, and they are trying to apply only memory learning which is very obscure and inadequate to say the least. We are justified in saying that the seven students who did not answer the question at all had no concept whatsoever of the term "ratio".

Question 8: The average of 16 ft., 18 ft., 24 ft., 26 ft., and 30 ft. is......

TABLE XXXIV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	1	0
Chose answer a. 24 ft.	12	4
Chose answer b. 22.8 ft. (correct)	233	78
Chose answer c. 114 ft.	40	13
Chose answer d. 57 ft.	9	3
Chose answer e. 20 ft.	5	2

Question 8 was one of the easiest problems on the examination paper. 78% chose the correct answer. The vast majority of the 300 students understood the meaning of "average" and were able to apply the routine of finding the average, to a particular problem. The group of 40 students, had one of two reasons for choosing answer "c". Either they confused the meaning of average with "sum" or else they summed the five numbers which is the first step in obtaining the average, but they then forgot to divided the sum by 5, the number of numbers being added. Those choosing answer "a" erroneously thought that the median of a group of numbers (the middle number to them) would be the average of the group. There is no logical justification for choosing answer "e", other than it being a guess on the part of the 5 students concerned.

Question 9: The area of a rectangle is x sq. in. If each side were made 5 times as long, the area would be........

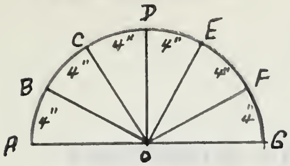
TABLE XXXV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, % ERRORS

	N	%
No Attempts	1	0
Chose answer a. 5x sq. in.	121	40
Chose answer b. 10x sq. in.	8	3
Chose answer c. 15x sq. in.	13	4
Chose answer d. 20x sq. in.	66	22
Chose answer e. 25x sq. in. (correct)	91	30

Question 9 is a straight forward question involving the relationship between the area of a rectangle and its two sides, yet only 30% of these students were able to chose the correct answer. The 121 students choosing answer "a" either misread the question and thought only one side was increased 5 times, or else they did not understand the functional relation between the sides of a rectangle and its area. There is no mathematical logic as to why 13 persons would choose answer "c", and as for the 66 choosing answer "d", they might have reasoned that a rectangle has four sides, and if each side is increased 5 times, then the new area would be 20x sq. in. This only goes to show again that the great majority of these 300 students do not understand the basic relationship of the area of a rectangle to its sides.

Question 10:



ABCDEFG is a semicircle. How large is each angle at O?

TABLE XXXVI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	2	1
Chose answer a. 10 degrees	6	2
Chose answer b. 15 degrees	12	4
Chose answer c. 30 degrees (correct)	264	88
Chose answer d. 45 degrees	7	2
Chose answer e. 60 degrees	9	3

Questinn 10 was well done. The 12 students choosing "b" apparently mistook the angle at "O" in the semicircle to be 90 degrees instead of 180 degrees, and thus dividing by the number of sectors, they would get answer "b" instead of answer "c". In a similar way, those choosing answer "e" apparently mistook the size of angle O as 360 degrees rather than 180 degrees.

Question 11: Which value of 'a' satisfies the equation:

$2(a-5) = 3(1-a)$?

TABLE XXXVII

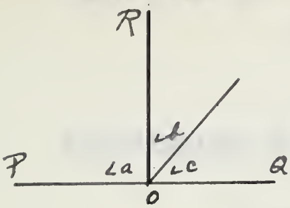
FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	21	7
Shose answer a. 5	43	14
Chose answer b. 2.6 (correct)	122	41
Chose answer c. 5/13	45	15
Chose answer d. 2 2/3	50	17
Chose answer e. 4 1/3	19	6

Only 41% of these students were able to get question 11, which indicates some difficulty for the majority of students. The technical expression, "Satisfies the equation" appears absolutely meaningless to the 21 students who made no attempt to solve the problem, and this is astonishing when one realizes how often that phrase must have been repeated in the class-room through-out the year. It would be interesting to know just how many students who were classed as D students made no attempt to solve the problem above. The remaining 52% making errors appear well distributed among the four wrong answers.

Had these students substituted their chosen value in the equation then solved for "a", they would have known for sure whether their answer was right or wrong.

Question 12:



POQ is a straight line and RO $\perp$ PQ. Which of the following equations is not necessarily true?

TABLE XXXVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	1	0
Chose answer a. $\angle a + \angle b + \angle c = 180$ degrees	15	5
Chose answer b. $\angle a = \angle b + \angle c$	27	9
Chose answer c. $\angle b + \angle c = 90$ degrees	18	6
Chose answer d. $\angle b = \angle c$ (correct)	232	78
Chose answer e. $\angle a = 90$ degrees	7	2

Question 12 was very well done. Those students choosing the wrong answer either misread the question as, "Which of the following equations IS necessarily true?", or else the negative wording of the question was too difficult for them to master.

Question 13: The sum of (+9), (-15), (-21), (-6), (+16) and (+4) is.....

TABLE XXXIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	4	1
Chose answer a. -73	11	4
Chose answer b. -23	29	10
Chose answer c. -13 (correct)	242	81
Chose answer d. +13	6	2
Chose answer e. +73	8	3

Question 13, a simple question, was well done. Answers other than the correct one were either due to guessing on the part of the students or extreme carelessness in handling signed numbers. It is possible that the 6 students who chose answer "d" had the correct answer "-13", then made a mistake in scoring the answer sheet by marking +13 instead.

Question 14: The five interior angles of a pentagon are together equal to.........

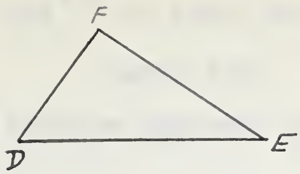
TABLE XL

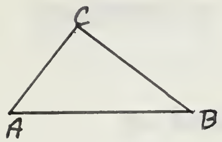
FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	0	0
Chose answer a. 360 deg.	120	40
Chose answer b. 450 deg.	34	11
Chose answer c. 500 deg.	2	1
Chose answer d. 540 deg. (correct)	99	33
Chose answer e. 720 deg.	45	15

Only 1/3 of our 300 students chose the correct answer on this problem. With this particular question it was difficult to tell whether the wrong answers were due to incorrect reasoning or guessing. Answer "a" could be chosen if one reasoned that a pentagon is made up of only two triangles, each containing 180 degrees; but it is incredible that 120 of these B standing students- 40% of the group, could reason in such a way especially if they first drew a rough sketch of the required figure. It is very likely that the 67% who chose the incorrect answers did not understand the concept of pentagon, and guessed at the probable answer.

Question 15:





ABC and DEF are two similar triangles. AC = 5", CB = 8", DF = 7". The length of FE is......

TABLE XLI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS,& ERRORS

	N	%
No Attempts	1	0
Chose answer a. 10"	191	64
Chose answer b. 10 1/5"	8	3
Chose answer c. 11"	25	8
Chose answer d. 11 1/5" (correct)	51	17
Chose answer e. 12"	24	8

This question was very poorly answered, and reveals two major weaknesses: first, the student's lack of understanding of similar triangles, and secondly, their weakness in handling problems dealing with ratio- a weakness previously revealed in question 7. There is only one justifiable reason why 64% of these students chose answer "a". They easily found the ratio of AC to CB as 5/8, and they evidently noticed the difference between the numerator and denominator as 3. To them is was logical that DF to FE would then be 7/10, which also shows a difference of three, between the numerator and denominator. The fact that 64% of these 300 B standing students reasoned in such a manner is proof that little logical thought entered into the solution of problems dealing with fractions and ratio.

Question 16: $(-2a^3)^2$ is equal to.......

TABLE XLII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	0	0
Chose answer a. $-2a^6$	58	19
Chose answer b. $-4a^5$	94	31
Chose answer c. $+4a^6$ (correct)	95	32
Chose answer d. $+4a^5$	32	11
Chose answer e. $+2a^6$	21	7

Only 32% of these students were able to compute correctly the answer to this question, which means that 68% do not understand either the rule of exponents for multiplication, which occupies two pages in the text, or they do not know how to handle coefficients within parentheses. The 58 students choosing answer "a", and the 21 students choosing answer "e" presumably understood the rule of exponents for multiplication, but they did not know how to handle signed numbers within parentheses. The 94 persons answering "b" understood neither the rule for multiplying exponents nor the squaring of signed numbers within parentheses. The 32 students choosing "d" correctly expended the squaring of the signed number, but they did not understand the expansion of exponents for multiplication. All in all, we are lead to believe that signed numbers and exponents are either very poorly taught in grade IX, or else the calibre of some of our future university students is very low in regards to the fundamental operations in mathematics.

Question 17: Which of the following equations is true?

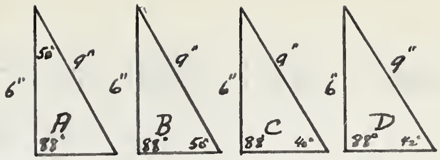
TABLE XLIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	10	3
Chose answer a. $\frac{1}{2} = \frac{3}{4}$	6	2
Chose answer b. $\frac{2\frac{1}{2}}{3} = \frac{4}{5}$	23	8
Chose answer c. $\frac{5}{6} = \frac{12}{10}$	41	13
Chose answer d. $\frac{3}{4} = \frac{2\frac{1}{2}}{3\frac{1}{2}}$	47	16
Chose answer e. $\frac{5}{6} = \frac{2\frac{1}{2}}{3}$ (correct)	173	58

The 53 students answering "a" and "d" are very likely part of the 191 students who in question 15 thought that if the difference between the numerator and the denominator of two fractions is the same, then the fractions must be equal. It was likely carelessness on the part of the 41 students answering "c" that they did not notice that the right hand side of the equation had to be reversed if it were to be the right answer, or else they did not understand the process of the L.C.D. in testing the equality of fractions.

Question 18:



Four triangles (A), (B), (C), and (D) have dimensions as shown in the figures above. Which pair of triangles is congruent?

TABLE XLIV

FREQUENCIES OF CORRECT ANSWERS , NO ATTEMPTS, & ERRORS

	N	%
No Attempts	1	0
Chose answer a. A and B	129	43
Chose answer b. A and C	31	10
Chose answer c. B and C	4	1
Chose answer d. B and D	14	5
Chose answer e. A and D (correct)	121	40

Only 40% at the most, including those that guessed correctly, chose the right answer. This means that at least, 60% of the students did not understand the meaning of congruence. It seems that the 129 students choosing answer "a" thought that as long as the three angles of one triangle were equal to the three angles of another triangle, irrespective of their not being corresponding angles, that the two triangles would be congruent. If they had realized that congruence implies equality of corresponding parts, then they would have had no difficulty in selecting the pair of congruent triangles.

Question 19: The product of $(-x)$, $(+2y)$, $(-3y)$, $(-4y)$, $(-5x)$ and $(+6y)$ is.......

TABLE XLV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	11	4
Chose answer a. $-720xy$	16	5
Chose answer b. $+720xy$	19	6
Chose answer c. $-720x^2y^4$	107	36
Chose answer d. $+720x^2y^4$ (correct)	136	45
Chose answer e. $+720x^3y^3$	11	4

Only 45% of these students were able to get the right answer to a straight forward question involving signed numbers and exponents. The 46 students answering "a,b,e," had no idea at all regarding the multiplication of monomials. The 107 students who answered "c" were able to handle the exponents properly, but made a mistake either in the handling of the negative numbers, or else they made an error in marking the answer on the score sheet.

Question 20: How many "inches per second" are equivalent to "20 ft. per minute"?

TABLE XLVI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	7	2
Chose answer a. 1200	71	24
Chose answer b. 240	75	25
Chose answer c. 1/3	53	18
Chose answer d. 4 (correct)	84	28
Chose answer e. 3/5	10	3

Question 20 was one of the most difficult of the 40 questinns in section A. The proper solution requfred two steps; first changing feet per minute to feet per second, and secondly, changing feet per second to inches per second. It seems very difficult for these students to do a problem involving more than one step, and if two or more steps are involved, the majority of these students are not able to think that far ahead and arrive at the proper solution. One moment of logical thought would have told the students answering "a and b" that their solutions were absurd, yet almost 50% of the 300 students in this investigation chose one of these two answers. Those answering "c" were on the right path, and got the first step, but they either forgot or did not realize that they were to change feet per second into inches per second, and they thus chose the answer 1/3.

Question 21: For which one of the following propositions is the converse necessarily true?

TABLE XLVII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS & ERRORS

	N	%
No Attempts	2	1
Chose answer a. If I am sick, I do not go to school.	65	22
Chose answer b. Triangles on the same base and between the same parallels are equal.	61	20
Chose answer c. Triangles which have corresponding sides equal, have corresponding angles equal.	54	18
Chose answer d. The angles at the base of an isosceles triangle are equal. (correct)	71	24
Chose answer e. Angles which are vertically opposite are equal.	47	16

The even distribution of answers makes us suspect that the great majority of students guessed at the chosen answer. The main reason for guessing being that the students were unable to formulate converses for each of the five given propositions. If no converse could be reasoned, then there was only one thing to do- guess. This question, like question 4, requires good verbal reasoning, and so far we have seen that the majority of these 300 students are a long way from being able to reason verbally with any degree of logical thought.

Question 22: The diameter of the base of a cylinder is 6 inches and its height is 20 inches. The volume is......

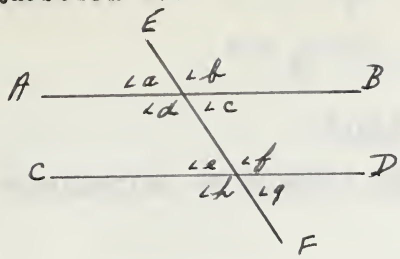
TABLE XLVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	19	6
Chose answer a. πrh or 377 1/7 sq. in.	82	28
Chose answer b. πr^2h or 2262 6/7 sq. in.	60	20
Chose answer c. πr^2h or 565 5/7 sq. in. (correct)	112	37
Chose answer d. πrh^2 or 7542 6/7 sq. in.	11	4
Chose answer e. πrh^2 or 3771 3/7 sq. in.	16	5

Question 22 shows that the great majority of these students are not able to handle the techniques or tools taught in the mathematics course in grade IX. This question is a straight application of a formula to a particular situation, yet, in the particular situation, these students are not able to apply the rules they learned in the class-room. Criticism of the form of this question will be left till a further chapter.

Question 23:



The two parallel lines AB, CD are cut by the transversal EF. Which one of the following equations is not true?

TABLE XLIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS,& ERRORS

	N	%
No Attempts	0	0
Chose answer a. $\angle d + \angle h = 180$ deg. (correct)	138	46
Chose answer b. $\angle d + \angle g = 180$ deg.	122	41
Chose answer c. $\angle c + \angle d = 180$ deg.	10	3
Chose answer d. $\angle b + \angle c = 180$ deg.	22	7
Chose answer e. $\angle a + \angle b = 180$ deg.	8	3

Care had to be taken in this question that the not true equation was selected, and this might have caused some reading difficulty for many of the students. If the students had known Theorem 8, they would have immediately realized that "a" was wrong- thus NOT true, and thus, the required answer. It is very likely that many errors were due to students forgetting to chose the incorrect equation. The 122 persons chosing "b" simply did not understand the meaning of corresponding angles.

Question 24: If $\pi = \frac{22}{7}$, r = 2.5" and h = 6.2"; then $V = \frac{1}{3}\pi r^2 h$ is equal (approximately) to..

TABLE L

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	25	8
Chose answer a. 20.9 cu. in.	30	10
Chose answer b. 22.9 cu. in.	38	13
Chose answer c. 30.5 cu. in.	47	16
Chose answer d. 32.5 cu. in.	53	17
Chose answer e. 40.6 cu. in. (corr.)	107	36

This question is a case of applying abstract theory to a concrete situation. Results- only 36% of these students were able to choose the right answer. The fact that there were 25 students who did not attempt the question reflects the complete inability of at least 8% of this group to egen understand the form of the problem. The scattered results in the first four suggested answers, is either indicative of much guessing or very poor computation on the part of 56% of the 300 pupils concerned.

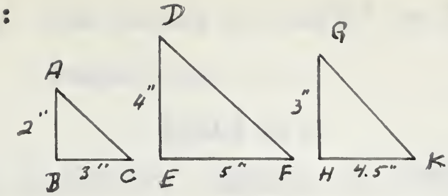
Question 25: Which has the greatest value?

TABLE LI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	5	2
Chose answer a. $\frac{7}{10} \times \frac{4}{3}$ (correct)	105	35
Chose answer b. $\frac{6}{5} - \frac{1}{3}$	29	10
Chose answer c. $\frac{8}{9}$	79	26
Chose answer d. $\frac{2}{3} + \frac{1}{4}$	47	15
Chose answer e. $\frac{3}{4} \div \frac{5}{6}$	35	12

Here, as in question 17 we have evidence that the students cannot handle fractions properly, whether it is addition, subtraction, multiplication, or division. The students in this investigation do not realize that the one satisfactory method of comparing fractions is to first reduce all the fractions to the lowest common denominator, then carry out comparisons.

Question 26: 

ABC, DEF and GHK are three right-angled triangles. The lengths of two sides of each triangle are as indicated in the figures. Which of the equations is not true?

TABLE LII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	5	2
Chose answer a. $\angle A = \angle G$	31	10
Chose answer b. $\angle C = \angle F$ (correct)	13	4
Chose answer c. AB/BC = GH/HK	95	32
Chose answer d. $GK/AC = \frac{3}{2}$	95	32
Chose answer e. $DF = \sqrt{41}$"	61	20

Question 26 appears to be the hardest and is certainly the most poorly done of the multiple choice questions. As in question 15, we have a straight forward question involving similar triangles and ratio- actually a very simple question- yet 96% of these students are unable to see the relationship between two similar figures, and the absence of relationship with a third unlike figure. The students are asked to pick out the equation which is not true, yet 94% of the students picked out the equations which were true. This could possibly be a result of poor reading ability on the part of quite a number of these students.

Question 27: One factor of $15a^8b^6$ is $3a^2b^3$. The other factor is......

TABLE LIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	4	1
Chose answer a. $5a^6b^3$ (correct)	92	31
Chose answer b. $5a^6b^2$	12	4
Chose answer c. $5a^4b^2$	62	21
Chose answer d. $5a^4b^3$	56	19
Chose answer e. $12a^6b^3$	74	24

As in questions 16 and 19, so here we see a weakness on the part of almost 70% of these students in the handling of exponents. 62 students divided the exponents of the one known factor into the product to get the answer "c". At least these students were consistent- they divided both exponents of the one factor into the product. The inconsistency of 48 students is that they divided by one of the exponents in the known factor, yet subtracted the other exponent in the very same factor. The strangest group is the group choosing "e". They understood the operations of exponents, but for some unknown reason, subtracted the one known numerical coefficient from the product, instead of dividing it into the product. .

Question 28: If $2(4n+1) = 3(5n-2)$, which of the statements is true?

TABLE LIV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	17	6
Chose answer a. $7n = 5$	50	17
Chose answer b. $7n = 16$	10	3
Chose answer c. $7n = 17$	13	4
Chose answer d. $7n = 24$	15	5
Chose answer e. $7n = 8$ (correct)	195	65

Question 28 was quite well done, yet many mistakes were made simply because the majority of students would not check their work. It would be interesting to know why 17% of these students chose "a". Answers "b,c,d," can be attributed mainly to guessing.

Question 29: A and B are two points 7 inches apart on a plane. The locus of all points on that plane that are 4 inches from A and 6 inches from B is.....

TABLE LV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	8	3
Chose answer a. One point	29	10
Chose answer b. Two points(correct)	81	27
Chose answer c. Three points	31	10
Chose answer d. A circle	80	26
Chose answer e. A straight line	71	24

The fact that only 27% of these students had the right answer is indication that the topic of loci is no better understood, by this grade of students, than many of the other basic topics in the grade IX mathematics course. We may reasonably conclude that the 70% who did not get the correct answer, chose their answer mainly by guessing. Had they made a rough sketch of the problem, they could not help but see that the required answer was none but "b". These poor results is also proof that the majority of the students concerned cannot visualize the problem they are trying to solve. They are using no logic in their thinking- they are depending on memory learning, which for them is very inaccurate to say the least. It would also be valuable to know,(but impossible to determine) how many students did not know the meaning of the word "locus" and thus guessed at one of the five suggested answers.

Question 30: What number, whole or fractional, bears the same ratio to 1 1/3 that 3 bears to 8?

TABLE LVI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	22	7
Chose answer a. 1/6	33	11
Chose answer b. $\frac{1}{2}$ (correct)	36	12
Chose answer c. 4/9	92	31
Chose answer d. 4	66	22
Chose answer e. 10 2/3	51	17

As to be expected from previous questions, any test item dealing with fractions and ratio is bound to show very poor results. This question is the third lowest of the first forty questions in regard to the percentage of pupils having the correct answer. These results compare favorably with those of questions 15 and 26 which also deal with ratio. If the 117 persons marking "d and e" had recognized the relationship between 3 and 8, they would have seen immediately that the number they were looking for would have to be less than a whole number, and thus answers "d and e" would be out of the question. But these students just do not realize that such rough estimations will often show the absurdity of many choices which they might make. It is our duty as teachers to continually emphasize this one very important mathematical routine.

Question 31: When $ab^4-3a^3b^3-5a^4b^2$ is divided by $(-ab^2)$, the quotient is....

TABLE LVII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	20	7
Chose answer a. $b^2+3a^2b-5a^4$	36	12
Chose answer b. $-b^2-3a^2b+5a^3$	37	12
Chose answer c. $-b^2-3a^2b+5a^3$	35	12
Chose answer d. $-b^2+3a^2b+5a^3$ (correct)	125	42
Chose answer e. $b^2+3a^2b+5a^4$	47	15

Question 31 was answered somewhat better than question 27 which also dealt with exponents. In all cases, the errors made can be classified under one of two main types- first errors in division of signed numbers, and secondly, errors in dividing by factors involving exponents. Both confirm previous statements that signed numbers and exponents are very poorly understood by the majority of students.

Question 32: To find the volume of a cylinder...

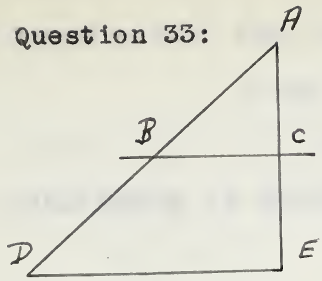
TABLE LVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	4	1
Chose answer a. Multiply the perimeter by the height.	13	4
Chose answer b. Multiply the area of the base by the height. (correct)	154	51
Chose answer c. Multiply the square of the radius by the height.	54	18
Chose answer d. Multiply the square of the radius by $\frac{22}{7}$	60	20
Chose Answer e. Multiply the cube of the radius by the height.	15	5

Only 50% of these students can verbalize a formula. 4% confused the formula for the volume of a cylinder with the formula for the area of the curved surface of a cylinder. This question, along with many others shows the inability of many of these 300 students to apply abstract or verbal reasoning to a particular problem, with any degree of success.

Question 33:



In the given figure BC//DE and AE ⊥ DE. If BC = 7", BD = 10" and DE = 13", then the length of CE is...

TABLE LIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	12	4
Chose answer a. 6½"	101	34
Chose answer b. 7"	97	32
Chose answer c. 7½"	26	9
Chose answer d. 8" (correct)	40	13
Chose answer e. 8½"	24	8

Question 33 was the fourth most poorly answered questions of section A. This problem involves several steps: first the necessity of dropping a perpendicular from B to DE, and secondly being able to see that the line so constructed became the third side of a right-angled triangle, whose sides were in the ratio of 3:4:5:. Lacking of insight into the relation of the Pythagorean theorem to the triangle newly formed was the main reason for only 13% of these students being able to chose the correct answer. The 97 pupils choosing answer "b" presumably thought that the perpendicular so dropped completed the formation of a square whose side CE would then be the same size as the side BC. These people were making a false assumption based on no logical thought.

Question 34: How long is each side of a cube that has a volume of 512 cu. in.?

TABLE LX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	21	7
Chose answer a. 6"	27	9
Chose answer b. 7"	23	8
Chose answer c. 8 (correct)	171	57
Chose answer d. 9"	40	13
Chose answer e. 10"	18	6

The proper solution of this problem rests on the ability of the students to remember the simple formula for the volume of a cube, then once realizing this formula, being able to calculate the cube root of the volume, or else substituting the suggested answers in the formula to see if the given volume were obtained. It is a pure case of applying a simple routine to a previously learned concept. The results show that only a little over half of the students were able to do so. Certainly those making an incorrect choice of answers did not substitute their values in the formula, as a positive check on the answer. If they did, it is the strangest coincidence that their multiplication would give the correct volume of 512 cu. in.

Question 35: $\frac{1}{4}$ % of 16 is......

TABLE LXI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	4	1
Chose answer a. .04 (correct)	117	39
Chose answer b. .4	42	14
Chose answer c. 4	112	37
Chose answer d. 40	7	2
Chose answer e. 64	18	6

Question 35 is a simple question involving percentage, yet the moment that a fraction enters into the calculation of a problem, there is utter confusion. 37% of these students ignored the percent sign and found $\frac{1}{4}$ of 16 instead of $\frac{1}{4}$% of 16. Answer "b" is either the result of carelessness, or weakness in handling decimals, but at least these students knew the correct procedure in using the % sign. The 25 students marking answers "d and e" had absolutely no idea of the meaning of $\frac{1}{4}$% else they would have seen that the answer could not possibly be larger than the number 16 itself.

Question 36: By what per cent does the ratio $\frac{5}{4}$ exceed the ratio $\frac{6}{5}$?

TABLE LXII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	21	7
Chose answer a. 4 1/6 (correct)	41	14
Chose answer b. 5	114	38
Chose answer c. 10	65	22
Chose answer d. 16 2/3	35	12
Chose answer e. 25	24	8

Again we see that a question involving fractinns, ratio, and percent is bougd to make a poor showing. This problem is certainly no exception. The correct solution involves three steps: first, changing both fractions to a common denominator, secondly, finding the difference between the two fractions, and thirdly, expressing this difference as a percentage of 6/5. The insight required to see these three steps belongs to a very small percent of these 300 students, and this percentage would probably be reduced greatly, if the students making fortunate guesses were subtracted.

The 114 students answering "b" misinterpreted the question. They did both step one and step two, and then expressed this fractional difference as a % of 100, not of 6/5.

Question 37: The area of a sector of a circle varies directly as....

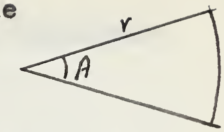


TABLE LXIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	17	6
Chose answer a. A	39	13
Chose answer b. r^2	20	7
Chose answer c. rA	71	23
Chose answer d. r^2A (correct)	120	40
Chose answer e. r^2A^2	33	11

It is likely that the majority of students in our particular group knew the formula for the area of a circle, but when it came to applying this relationship to a particular problem, there were only 120 out of the 300 students that had the correct answer. Actually, answers "a,b,d" are also correct, because the area of a sector of A circle does vary according to these three factors. Only when one stresses THE particular circle, is answer "d" the only truly correct answer. The 104 persons answering with "c and e" had no idea of the relationship between the area of a sector of a circle and the area of a circle, else these two answers could not possibly have been chosen.

Question 38: If $\frac{y-1}{3} = \frac{1}{4}(y+4)$, which of the equations is true?

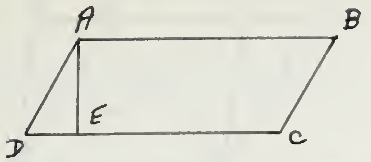
TABLE LXIV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	21	7
Chose answer a.		
$3(y+4) = \frac{1}{4}(y-1)$	88	29
Chose answer b.		
$4(y-1) = 1/3\ (y+4)$	23	8
Chose answer c.		
$12(y-1) = 3(y+4)$	63	21
Chose answer d.		
$\frac{1}{4}(y-1) = 1/3\ (y+4)$	47	16
Chose answer e.		
$4(y-1) = 3(y+4)$ (correct)	58	19

Again we see the small percentage of students getting the correct answer to a problem which involves fractional coefficients. 81% could not multiply through by 12 and clear the equation of fractions. 37% are also very inconsistent in that they multiply one side of an equation properly, yet make a major blunder in multiplying on the other side of the equation. This is seen in the answers "a,b". There is no reason, other than guessing, for the remainder of the errors made.

Question 39:



ABCD is a parallelogram. AE ⊥ DC. If AB = $4\frac{1}{2}$", AD = $2\frac{1}{2}$" and DE = $1\frac{1}{2}$", then the area of the parallelogram ABCD is......

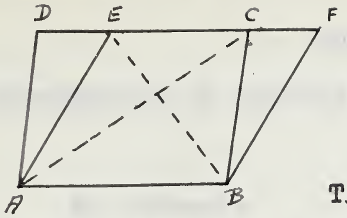
TABLE LXV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	23	8
Chose answer a. 7 sq. in.	29	10
Chose answer b. $7\frac{1}{2}$ sq. in.	44	15
Chose answer c. 8 sq. in.	23	8
Chose answer d. $8\frac{1}{2}$ sq. in.	74	25
Chose answer e. 9 sq. in. (correct)	107	36

Only 1/3 of these students chose the correct answer to a rather easy two-step problem. It is almost impossible to believe that 74 students would add the three given numbers, and take this sum as the area of the parallelogram, yet that is the only way that answer "d" could have been obtained, other than by guessing. 10% presumed that the area of a parallelogram is the sum of two adjacent sides, and proceeded to add inches to inches and in some manner got square inches in the answer.

Question 40:



In the given figure DF//AB and AB = DC = EF. Which of the statements is not true?

TABLE LXVI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	9	3
Chose answer a.		
Area //m ABCD = Area //m ABFE	24	8
Chose answer b.		
Area $\triangle$ ABC = Area $\triangle$ AEB	39	13
Chose answer c.		
Area //m ABCD = 2x area $\triangle$ ADC	57	19
Chose answer d.		
Area $\triangle$ BCF = $\frac{1}{2}$BCxCF (correct)	160	53
Chose answer e.		
AE = BF	11	4

Question 40 is evidence that 47% of this group does not know the basic proofs of the grade IX course in mathematics- especially theorems 20,21, and 22. If they did know these three particular theorems, plus theorem 17 it would only have been a matter of elimination of the true statements to find the incorrect answer.

Question 41: Simplify: $1/5x + 5/6x - \frac{1}{2}x$

TABLE LXVII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
No Attempts	6	2
Correct answer 8/15x	127	42
Omitted coefficient x ex. 8/15	35	11
$\frac{1}{2}$x	27	9
Mechanical errors in collecting terms, after having L.C.D.	21	7
1 8/15x	18	6
16x	17	6
1 1/5x	16	5
8x	9	3
11/30x	8	3
1/5	5	2
Errors in signs ex. -8/15x	3	1
Other errors	8	3

The main reason for such poor results on a very simple question is that the majority of pupils just do not understand the fundamental operations of fractions. It is almost impossible for them to correctly calculate the L.C.D. of several fractions. We may assume that it is mainly carelessness that made 35 students omit the "x" coeffieient, although some of these likely did not realize the necessity of the literal coefficient in the answer. The 27 persons with answer $\frac{1}{2}$x presumably mistook 1/5x for 1/6x, added it to 5/6x, subtracted $\frac{1}{2}$x and got $\frac{1}{2}$x.

Those students with the answer 16x correctly worked out the answer to the last step but either forgot the denominator 30, or else they did not realize that it was part of the answer. Broadly speaking, this question reflects previous statements that problems involving fractions are usually quite poorly answered.

Question 42: Simplify: $1.4 \div 4.2$

TABLE LXVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer 1/3	96	32
No attempts	10	3
Used divisor as dividend	92	31
Misplaced decimal	40	13
Mechanical errors in division	33	11
Insufficient significant figures	12	4
Other errors	17	6

It is surprising that a question based on grade 7 and 8 mathematics should be done correctly by only 32% of the 300 grade IX pupils concerned with this investigation. The most frequent type of error was that of confusing the divisor with the dividend, and this fact combined with the 40 students who made errors in placing the decimal point, show just how weak these students are in their fundamental operations.

Question 43: Simplify: 5/6X 3 1/3

TABLE LXIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer 25/9 or 2 7/9	186	62
No attempts	5	2
Mechanical errors in multiplication and division	41	14
Misread question ex. 5/8X3 1/3	15	5
Multiplied by 5 instead of 5/6	12	4
Multiplied by 10 instead of 10/3	9	3
Inverted coefficient 5/6	9	3
Inverted mixed fraction 3 1/3	7	2
Errors in decimals	4	1
Other errors	12	4

This simple question involving multiplication of fractions was quite well done, yet it is surprising that only 62% of these students got the right answer. It is understandable that 14% might make errors in the fundamental operations, but there should not be 12% losing full marks for careless reading of the question. This point must be continually stressed when teachers are preparing students for any examination- the importance of reading correctly and doing what the question asks.

Question 44: Simplify: $4/5 \div 9/10$

TABLE LXX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer 8/9	226	75
No attempts	7	2
Mechanical errors in multiplication and division	34	11
Inverted fraction 4/5 and then multiplied	14	5
Multiplied fractions instead of dividing	5	2
Misread numbers	4	1
Subtracted fractions	4	1
Other errors	6	2

Question 44 involves a similar technique to that of question 42, yet here we see 75% of these students getting the correct answer for question 44 and only 32% getting the correct answer for question 42. The main source of error in this question was in the mechanics of multiplying and dividing. Only 5% confused the divisor with the dividend in question 44.

Question 45: From $6a^3 - 5a^2 + 3a - 14$ subtract
$2a^3 - 4a + 7a^2 - 17$

TABLE LXXI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer $4a^3 - 12a^2 + 7a + 3$	67	32
No attempts	7	2
Subtracted before subtrahend arranged in descending order.	58	19
Some terms added instead of being subtracted.	51	17
Signs not changed to subtract	38	13
Mechanical error in subtraction and addition	35	12
Errors in exponents	29	10
Other errors	15	5

Question 45 was poorly done. Only 67 persons had the correct answer, one person was given one mark, and 232 persons got zero marks. This question also reveals the lack of basic fundamentals with regards polynomials. There were 58 students subtracting without first rearranging the subtrahend in descending order. The fact that 30% lost full marks because they could not handle signed numbers correctly supports the statements made regarding questions 19 and 31, which also involve handling of signed numbers.

Question 46: Divide $20a^3-2-27a^2+18a$ by $(5a-3)$.

TABLE LXXII

FREQUENCIES OF CORRECT ANSWERS , NO ATTEMPTS, & ERRORS

	N	%
Correct answer $4a^2-3a+1$, remainder $4a+1$	58	19
No attempts	48	16
Mechanical error in division and multiplication	45	15
Mechanical error in addition and subtraction	28	9
Incomplete division ex. $(4a^2-3a)$	34	11
Terms not arranged in descending order before operations carried out.	30	10
Errors in signs	26	9
Errors in exponents	20	7
Other errors	11	4

Question 46 appeared very difficult for the majority of these students. 65% made attempts which netted no marks at all, and an additional 16% were not able to attempt the problem at all.

One might say that this question was somewhat unfair in that the students were faced with uneven division in the third division step. It is very unlikely that many of the grade IX students had been exposed to such a question, and when they were confronted with uneven division, it was beyond their ability to handle the required operations. Evidence of this is seen in those 34 students who had incomplete division. Other errors again reflect weakness in the use of signed numbers and polynomials.

Question 47: If $P = 2n^2 + 7n - 15$, $Q = 10n^2 - 29n + 21$, and $R = 2n - 3$, find $(P + Q) \div R$.

TABLE LXXIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	&
Correct answer $6n-2$	83	28
No attempts	60	20
Errors in exponents	40	13
Mechanical errors in division and multiplication	50	17
Mechanical errors in addition and subtraction	25	8
Errors in signs	26	9
Did not divide by R	6	2
Other errors	10	3

Question 47 is one of the harder problems on the paper. Only 3 other questions have a higher percentage of no attempts than question 47. 37 persons were given only one mark, one was given two marks, and 180 people received zero marks. The one mark was given only if $(P+Q)$ were added correctly, and the other two marks were given if the sum were correctly divided by R. The greatest number of errors was in the use of exponents. Almost all the errors here were due to the students adding the exponents as well as the numerical coefficients of the like terms. The other main group of errors was the result of carelessness in the fundamental operations.

Question 48: Simplify by removing brackets and collecting terms: $5a-3(a-4) + 2(1-a) - (2a-5)$

TABLE LXXIV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer 19-2a	76	25
No attempt	23	8
Mechanical errors in addition and subtraction.	65	32
Errors in removing parentheses	55	18
Errors in using signs	34	11
Terms not collected in answer	19	6
Plus sign interpreted as the equality sign, and equation solved for "a".	18	6
Errors involving exponents	9	3
Other errors	1	0

No partial marks were given for this question in which 67% received zero marks, and 8 percent made no attempt. The main errors were mechanical errors in the four fundamental operations, and also in removing of parentheses. From the results of previous questions it is not surprising to see 11% losing full marks because they cannot use signed numbers correctly.

Question 49: Simplify: $\dfrac{12m^6n^5 - 30m^8n^7}{6m^2n}$

TABLE LXXV

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer $2m^4n^4 - 5m^6n^6$ or $m^4n^4(2-5m^2n^2)$	110	37
No attempts	33	11
Divided exponents	61	20
Subtracted terms in numerator then divided by $6m^2n$	21	7
Divided $6m^2n$ into only one term of the numerator, then subtracted.	19	6
Mechanical errors in subtracting exponents	17	6
Mechanical error in dividing numerical coefficients	12	4
Added terms in numerator then divided by $6m^2n$	12	4
Omitted literal coefficients "m and n" in the answer	5	2
Errors in signs	5	2
Calculated answer and placed it over original denominator	3	1
Other errors	2	1

This question is quite similar in procedure to questions 27, and the results are very similar. Here we have 37% of the students getting full marks compared with 31% for question 27. 20% again show lack of understanding of the laws of exponents. The other types of errors are quite evenly distributed, with mechanical errors still the cause of many students being credited with zero marks.

Question 50: If $7/12K-\frac{1}{4}K = 2K-1\ 2/3$ find the value of K.

TABLE LXXVI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

	N	%
Correct answer $K = 1\frac{6}{}$	55	18
No attempts	81	27
Mechanical errors in transposing terms across equality sign.	55	18
Mechanical errors in multiplying	8	3
Mechanical errors in subtracting	22	7
Changed only part of the equation to L.C.D.	24	8
Omitted right hand side of equation in all calculations	8	3
Errors in signs	16	5
Misread question when beginning calculations	8	3
Multiplied terms instead of subtracting	5	2
Other errors	10	3

Question 50 was a very good test item. 11 persons got one mark for collecting "like terms", four persons got two marks for simplifying the like terms, no one got three marks, and 55 persons got full marks for the complete solution. Question 50 appears beyond the capacity of 27%, who made no attempt to solve for the value of K. The process of transposing accounted for 18% of the errors, mechanical errors in the fundamental operations accounted for another 10%, and again the inability to use signed numbers properly penalized 5% of the group of 300.

Question 51: A line parallel to the base B^C of a triangle ABC cuts the sides AB and Ac at D and E. Prove that ABE and ACD are equal in area.

TABLE LXXVII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

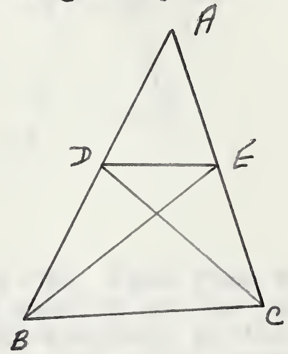
	N	%
Correct proof (full marks)	1	0
No attempts	38	13
Right diagram-wrong proof- no marks given	118	39
Wrong diagram with resulting wrong proofs	78	26
Right diagram- no proof	52	17
Incomplete proofs - partial marks given	13	4

This question was very poorly answered. Only one person got full marks (6 marks), twelve others received partial marks ranging from 2 marks to 5 marks. 52 others got the right diagram and correct construction and were given one mark each. No marks were given for the right diagram when the construction was not included. The 118 persons with the right diagram, and wrong proofs had absolutely no idea of the necessary proofs. One such exampbe will be seen on the following page. 78 students had the wrong diagrams, yet they tried, though unsuccessfully to prove that ABE and ACD areequal in area.

Sample 1

This is a proof from one of the 118 students who had the proper diagram, but had not stated the construction nor shown any insight whatsoever into the proper solution of this problem.

This sample proof is as exact a reproduction of the given proof as the writer could make. Nothing has been added or taken away from the original proof.



DE// to BC

AB = AC

DB = EC

AB = EC

AD = AE

Therefore ABC and ACD are equal in area by (theorem SAS or //)

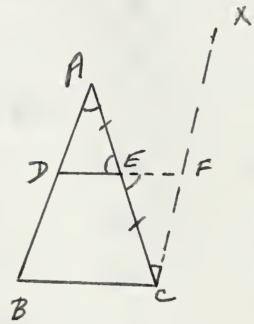
There are several points that should be explained in this solution which might account for the person getting zero marks. Firstly, he does not even write down what is given, or required to be proven, and he does not state that he joined DC to BE (thereby losing one mark).

Secondly, several false assumptions are made. He assumes A^b = AC, but he gives no proof for such assumption. He also assumes (falsely) that DB = EC, that AD = AE, and worst of all that AB = EC,

Thirdly, he says that ABC and ACD are equal by "Theorme" (note spelling) SAS, yet he has not even mentioned a pair of angles in his entire proof. This is a typical sample of the 118 proofs,- proofs that reflect no idea of the proper solution.

Sample 2

This is a proof given by one of the 78 students. Even though the diagram is entirely wrong, it is so surprising that these students can make-up a proof, which to them seems to be perfectly acceptable.



Given

Triangles ABC with D the midpoint of AB and E the midpoint of AC.

Req. to Prove

That ADE and EFC are equal in area.

Construction

From the point C draw a line CX. Then from the point E extend the line DE to a line CX meeting at the point F.
AE equals EC because E is the midpoint of the line AC. Angle CEF equals angle AED because vertically opposite angles are equal. Angle DAE equals angle ECF because they are corresponding angles in congruent triangles. Therefore triangle ADE equals triangle EFC.

Sample 2 above is typical of the 78 students that had the wrong diagram with resulting wrong proofs for question 51. The main difficulty with most of these 78 students is that they are not able to read a problem correctly or to construct a diagram to fit the problem. The above solution has absolutely no relationship with the problem given on the examination paper.

Question 52: Prove that the bisectors of the angles of a parallelogram form a rectangle.

TABLE LXXVIII

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

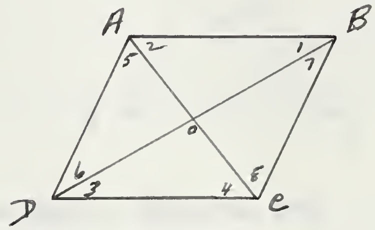
	N	%
Correct proof- full marks	0	0
Wrong diagram- wrong proof or no proof	182	61
Right diagram- no proof	28	9
Right diagram- wrong proof	18	6
Incomplete proof- partial marks	1	0

Question 52 was the most poorly answered of all the questions on the examination paper, with not one person getting full marks. There were 260 students in this group with a zero score, 39 were given one mark each for having the correct diagram, one person got two marks, and no one got more than two marks out of a possible total of 6 marks. 71 persons were unable to attempt the question- they could not even draw the diagram.

182 persons had wrong diagrams with resulting wrong proofs, a sample of which will be seen on the following page. 28 students were able to draw the correct diagram, but were able to give no construction or proof. 18 persons with the correct diagram attempted to prove the question asked, but their proofs reflect a complete absence of understanding and insight into the correct procedure and solution. Sample 4 is typical of this group.

Sample 3

The proof given below is typical of those given by the 182 students who had the wrong diagram for question 52, with resulting wrong proof, or no proof.



<u>Given</u>

A //gm ABCD

AB//DC

AD//BC

<u>To Prove</u>

AB = DC

AD = BC

<u>Proof</u>

<u>In triangles AOB and DOC</u>

Statements	Reasons
AB // DC	Given
angle 1 = angle 3	alternate angles
angle 2 = angle 4	alternate angles
Therefore △ ≡ △ DOC	A.S.A.
AB = DC	

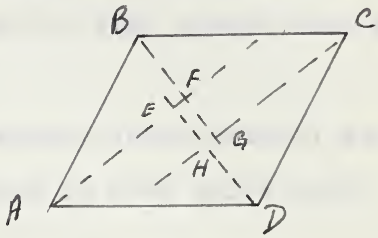
<u>In triangles AOD and BOC</u>

AD//BC	Given
angle 6 = angle 7	alternate angles
angle 5 = angle 8	alternate angles
Therefore	
△ AOD ≡ △ BOC	A.S.A.
AD = BC	

This student made one false assumption at the beginning of the problem. He assumed that the straight lines joining the opposite angles of a parallelogram were the bisectors of the angles of the parallelogram. Had this person read the question through properly, he would have seen that what was being proved was not what the question asked to be proved. The whole proof thus was meaningless.

Sample 4

Sample 4 is a copy of one of the proofs given by one of the 18 students who were able to construct the correct diagram, but geve a proof that had absolutely no relation to the problem.



Proof

BC//AD
---Given
AB//DC

angle CBA equals angle CDA -- Theorem 17B

angle BAD equals angle DCB -- Theorem 17B

BH bisects angle ABC
AF bisects angle BAD ---- constructions
DF bisects angle ADC
CH bisects angle DCB

So EFGH must be a rectangle.

Sample 4 reveals the major weakness in most students' work- that they will not write down what is given, what is required to be proved, and what the construction is. This student repeated what was given in the question, called this the proof, and concluded "so EFGH must be a rectangle". This student, a fair representative of the majority of these 300 students, knows neither proofs, nor constructions, and above all he applies little or no logical thought in solving his problems.

Question 53: Production of crude petroleum in Alberta and in Canada (to nearest 10,000 bbls.), 1933-45. (See graph on examination paper)

a. By what percentage (approximate) did Canada's production in 1938 exceed that of 1937?

b. By what percentage (approximate) was Alberta's production in 1943 below that of 1942?

c. Compare (as an approximate ratio) the total Canadian production during the four-year period, 1938-41, with the total production in the four-year period 1942-45.

d. What was Alberta's average annual production?

TABLE LXXIX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS (for 53a)

	N	%
Correct answer 130%-145%	7	2
No attempts	17	6
Various answers from 4/11 to 10	44	15
" " " 11 to 50	116	39
" " " 51 to 100	35	12
" " " 101 to 130	4	1
" " " 146 to 200	3	1
" " " 201 to 500	45	15
" " " 501 to 5000	4	1
" " " 5001 to 50000	8	3
" " above 50,000	7	2
Miscellaneous - ex. "yes", 40	10	3

Question 53 was one of the most poorly answered items on the examination, and part a was the second most poorly answered of the four parts, with only 7 students receiving credit of 2 marks each for part a. No partial marks were given- the students got either zero marks or two marks, for each of the four parts. It is easy to see that 86% of the students in this group of 300 know absolutely nothing about graphs. With the correct answer anywhere between 130% and 145%, we see only four persons that are anywhere near the correct answer, and these four had answers between 100% and 130%. Three others had answers between 146% and 200%. The range of answers was from a low of 4/11% to 4 million percent, and this fact in itself gives a fairly accurate picture of the understanding of graphs as held by the vast majority of these 300 students. One student answered "yes", and several others gave answers expressed in bbls., when the question distinctly asks for percentage. Common sense and a sense of reason, should have told many of the ridiculous answers they were writing. The fact that 15% had answers between 5000% and 500,000% shows the lacking of understanding in relation to the concept of percentage.

TABLE LXXX

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS

(for 53b)

	N	%
Correct answer 4% - 6%	40	13
No attempts	26	9
Various answers from .021 to 1	13	4
" " " 1 to 4	30	10
" " " 6.1 to 10	35	12
" " " 11 to 25	29	10
" " " 26 to 100	76	25
" " " 101 to 1000	11	4
" " over 1000	19	6
Other answers 6/10 to 1-/8; 15.74 930/955; ¼ bbls.	21	7

Although part b was done approximately 6 times as well as part a, there were still only 13% of the group receiving credit for the correct answer. 22% were within 4% of the correct answer which is a much better performance than that of part a, yet we still see a great number of very ridiculous answers, ranging from .021% up to 1,005,000%. Many did not read the question properly, and expressed the answer in "bbls.", when the questinn clearly asks for percentage.

TABLE LXXXI

FREQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS
(for 53c)

	N	%
Correct answer 33/39 approx.	21	7
No attempts	94	31
Various answers from $\frac{1}{10}$ to $\frac{80}{100}$	47	16
" " " $\frac{80}{100}$ to 2	11	4
" " " 2.1 to 4	12	4
" " " 4.1 to 10	8	3
" " " 11 to 100	7	2
" " " 101 to 500	35	12
" " " 501 to 1000	16	5
" " over 1000	23	8
Other answers 700/1018; 2 is to $1\frac{1}{2}$; 395/10,000 bbls. "up grade to a decline"	26	9

The results in part c compare favorably with parts a and b, with only 7% of these students finding the correct approximate ratio, and 31% making no attempt to solve the problem.

The range of marks is as extreme as in parts a and b, with answers from 1/10 to 10 million, and many answers expressed in terms of bbls. Again the results show a complete absence of understanding of graphs and ratio.

TABLE LXXXII

FRQUENCIES OF CORRECT ANSWERS, NO ATTEMPTS, & ERRORS
(for 53d)

			N	%
Correct answer		5,800,000 to 6,000,000 bbls.	3	1
No attempts			75	25
Various answers from		10 to 100	20	7
"	" "	101 to 500	29	10
"	" "	501 to 5000	82	27
"	" "	5001 to 50000	14	5
"	" "	50001 to 5800000	39	13
"	" "	6 million to 10 million	15	5
Other answers		207%; 75 gallons; 586 12/13%	14	5

Next to question 52, question 53d was the most poorly answered of all the examination questions. The fact that 20 students said that Alberta's average annual oil production was between 10 and 100 bbls. is sufficient evidence that almost all of these 300 students applied neither logical thought to the problem, nor gave a second thought about the meaning of their answer.

Conclusions regarding question 53

In analyzing question 53 as a whole, one sees a tremendous lack in the understanding of percentage, ratio, and graphical interpretation. It is true that the units used on the graph were the main source of confusion, and it may be said that inadequate preparation in the classroom for this type of graph was very likely a prime cause of so much confusion. Certainly this type of graph is much different from those graphs which the students experienced in the authorized text-book.

Table XVI on page 14 sums up the students' ability to understand graphs, and graphical concepts. There we see, out of a possible total of 8 marks, 234 persons with zero marks, 60 with 2 marks, 6 with 4 marks, and no one for 6 marks or 8 marks.

CHAPTER V

ANALYSIS OF THE TEXT BOOK

As mentioned in chapter I, the text book will be analyzed for: a. Relationships

b. Concepts, definitions or technical vocabulary

c. Routines or techniques.

Mr. French and Mr. Massing both had difficulty in defining each of the terms, "generalizations, language, and information", which were the subject matter of their theses. Similarly, with this thesis, there is such an overlap in meaning among the three terms, that no hard and fast definitions of the three can be formulated.

Katz and Schanck in their book 'Social Psycholggy' make the following statement, " To see identity in differences is the basis of relationships". Relationships and generalizations thus become very closely tied together. The above mentioned statement, and Webster's definition of a relationship as "the mode in which two or more things stand to one another" will be taken as the basic meaning of 'relationships'. All theorems, propositions and formulae will be classified as relationships.

Concepts in this thesis will be understood to include all the technical vocabulary and the appropriate definitions found in the text book for this technical vocabulary.

Routines or techniques are here defined as the application of facts or methods used in solving various problems. They are non-relational- they are specialization

on a particular problem- they are the tools with which the problems are solved. Example: finding the area of a square, simplification of an expression , constructing a certain figure, and solving a linear equation, are all routines or techniques. The knowledge that the area of a circle is "pi x r^2" is an expression of the relationship between the area of a circle and its radius. The application of this formula in finding the area of a particular circle is a routine or technique. The definition of a circle is a concept.

With these definitions in mind the text book has been analyzed for relationships, concepts, and routines.

A. <u>Analysis of the text for Relationships</u>

Some 81 relationships listed under 12 main headings have been discovered in the text. The 12 main headings have been arranged alphabetically, and all the relationships relative to that particular topic have been listed underneath each heading. In some cases a duplication of relationships will be notices as the particular relationship was relatige to more than one main topic. Example: The formula for the area of a circle will be found under three headings; area, circles, and formula. Because it IS relative to each of these three topics, it has been included in each as a relationship.

I. <u>Area</u>

1. Area of a circle equals πr^2.

2. Area of a circle varies with the radius squared.

3. Area of a rectangle: $A = LxW$.

4. Area of a //gm. equals the product of its base and its height.

5. Area of a triangle equals one half the base times its height.

6. Theorem 20: Parallelograms on the same base and between the same parallels (or of the same altitude) are equal in area.

7. Corollary 1a (Theorem 20): The area of a parallelogram is equal to the product of its base and its altitude.

8. Corollary 1b (Theorem 20): The area of a parallelogram is the same as that of a rectangle on the same base and between the same parallels.

9. Corollary 2 (Theorem 20): Parallelograms on equal bases and of the same altitude are equal in area.

10. Area of a square equals the product of any two of its sides.

11. Area of a trapezium equals one half of the height times the sum of the two parallel sides.

II. Angles

1. Theorem 6: When a straight line cuts two parallel straight lines, the alternate angles are equal.

2. Theorem 7: When a straight line cuts two parallel straight lines, the interior angles on the same side of the cutting line are together equal to two right angles.

3. If the sum of two angles is a right angle, the angles are called complementary angles.

4. Theorem 1: Complements of the same or of equal angles are equal.

5. If the sum of two angles is 180 degrees, the angles are supplementary and each is the supplement of the other.

6. Theorem 2: When one straight line meets another straight line, the sum of the adjacent angles is equal to two right angles.

7. Theorem 3: Supplements of the same angle or of equal angles are equal.

8. Theorem 4: If any number of straight lines meet at a point, the sum of all the adjacent angles is equal to four right angles.

9. Theorem 5: If two straight lines intersect, the vertically opposite angles are equal.

10. Theorem 9: The sum of the angles of a triangle equals 180 degrees or two right angles.

11. When any side of a triangle is produced in either direction it forms an exterior angle of the triangle.

12. Theorem 10: If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two opposite interior angles.

13. Theorem 8: When a straight line cuts two parallel straight lines, the corresponding angles are equal.

III. Converses

1. If we interchange the conclusion and the given part of the theorem, we have the converse of the theorem.

2. Theorems 6a: When a straight line cuts two other straight lines, if a pair of alternate angles

are equal, then the two straight lines are parallel.

3. Theorem 7a: When a straight line cuts two other straight lines, if a pair of interior angles on the same side of the cutting line are together equal to two right angles, then the two straight lines are parallel.

4. Theorem 8a: When a straight line cuts two other straight lines, if a pair of corresponding angles are equal, then the two straight lines are parallel.

IV. Circles

1. The area of a circle equals "pi x r^2".
2. The circumference of a circle equals "pi x d"

V. Formulas

1. Linear Formulae

   a. The perimeter of a rectangle $= 2(L + W)$, where L is the length and W is the width.

   b. The circumference of a circle equals 3 1/7 times the diameter,

   c. $C = 22/7\ d$

   d. $C = \pi d$

   e. $C = 2\pi r$, where "r" is the radius.

2. Area Formulae

   a. Area of a circle $= \pi r^2$

   b. Area of a //gm. equals the product of its base and its height. ($A = bh$)

   c. Area of a rectangle equals the

product of its length and its width. ($A = lw$)

d. Area of a square is the product of any of its two sides. ($A = s^2$)

e. Area of a trapezium is found by the formula $A = \frac{h}{2}(b+u)$, where 'h' is its altitude, 'b' is the lower base, and 'u' is the upper base.

f. Area of a triangle is equal to half the product of its base and its height. ($A = \frac{1}{2}bh$)

3. Volume Formulae

a. The volume of a cube is found by multiplying together the three equal members. ($V = e^3$) where 'e' is the length of the sides of the cube.

b. The volume of a cylinder is found by multiplying the area of its base by its height. ($V = \pi r^2 h$)

c. The volume of a prism may be found by multiplying the area of the base by the height. ($V = Bh$)

d. The volume of any rectangular solid is the product of the lenth, width, and height.

4. Other Formulae

a. Simple interest. $i = prt$, where 'i' is interest, 'p' is principal, 'r' is rate of interest, and 't' is time.

b. Distance. $d = rt$, where 'd' is distance, 'r' is rate of movement, and 't' is time.

c. Amount formula. Amount equals the principal plus the interest. ($A = p + prt$), where 'A' is amount, 'p' is principal, 'r' is rate of interest, and 't' is time.

VI. <u>Graphs</u>

1. When the graph of a mathematical formula is drawn, we are really picturing the relationship between changing quantities which the formula expresses.

VII. <u>Loci</u>

1. The locus of all points equally distant from two given points is the right bisector of the straight line joining the points.

2. The locus of all poimts that are equidistant from two intersecting straight lines is the bisector of the angles so formed by intersection.

3. The locus of a point that is equidistant from three given point that are not in a straight line is that point of intersection of the two right bisectors of the two lines so formed by joining the three points.

4. The locus of a point at a fixed distance from a given point is a circle.

5. The locus of a point equally distant from a given point is a sphere.

6. The locus of a point equally distant from a given line is a cylinder.

7. The locus of a point equally distant from two given points is a plane.

VIII. Pythagorean Theorem

1. In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

IX Parallel Straight Lines

1. When a straight line cuts two parallel straight lines:

a. the alternate angles are equal

b. the interior angles on the same side of the cutting line are together equal to two right angles

c. the corresponding angles are equal

2. When a straight line cuts two other straight lines:

a. if a pair of alternate angles are equal, then the two straight lines are parallel

b. if a pair of interior angles on the same side of the cutting line are together equal to two right angles, then the two straight lines are parallel

c. if a pair of corresponding angles are equal, then the two straight lines are parallel.

3. If three or more parallels intercept equal segments onone transversal, they intercept equal segments on every other transversal.

4. The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side.

5. The straight line joining the middle point of two sides of a triangle is parallel to the third side and equal to half of it.

6. The line which joins the mid-point of two adjacent sides of any quadrilateral is equal and parallel to the line which joins the midpoints of the other two sides.

X. Parallelograms

1. The area of a parallelogram is equal to the product of its base and its height.

2. The opposite sides of a parallelogram are equal.

3. The opposite angles of a parallelogram are equal.

4. Each diagonal bisects the parallelogram.

5. The diagonals of a parallelogram bisect each other.

6. If one angle of a parallelogram is a right angle, all the angles must be right angles.

7. If two adjacent sides of a parallelogram are equal, all its sides are equal.

8. Parallelograms on the same base and between the same parallels (or of the same altitude) are equal in area.

9. The area of a parallelogram is the same as that of a rectangle on the same base and between the same parallels.

10. Parallelograms on equal bases and of the same altitude are equal in area.

11. If a triangle and a parallelogram stand on the same base and between the same parallels, the area of the triangle is half that of the parallelogram.

XI. Similarity

1. When two figures are similar:

a. their corresponding angles are equal.

b. the ratio of any two corresponding sides equals the ratio of any other pair of corresponding sides.

XII. Triangles

1. The area of a triangle is equal to half the product of its base and its height.

2. The sum of the angles of a triangle is equal to 180 degrees.

3. The corresponding parts of two congruent triangles are those parts which exactly coincide when one figure is placed on top of the other.

4. If two triangles are congruent, their corresponding sides must be equal and their corresponding angles must also by equal.

5. If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, the triangles are congruent.

6. If two triangles have two angles and a side of one respectively equal to two angles and the corresponding side of the other, the triangles are congruent.

7. If two triangles have three sides of one respectively equal to three sides of the other, the triangles are congruent.

8. If two right angled triangles have the hypotenuse and one side of one equal to the hypotenuse and one side of the other, the triangles are congruent.

9. The angles at the base of an isosceles triangle are equal.

10. If the angles at the base of a triangle are equal, the sides opposite to these angles are equal.

11. The sum of the angles of a triangle is equal to two right angles.

12. If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two opposite interior angles.

B. Analysis of the text for Concepts

The following list comprises the concepts or technical vocabulary found in 'Mathematics for Today'. The terms are listed alphabetically, under main topics, and are to be considered separately from the relationships listed previously. This list comprises 303 technical terms, but the definitions of each, which are also a part of the classification of concepts, will not be included in this thesis.

The concepts found in the Algebra section of the text are intermingled with those found in the Geometry section, but all are arranged alphebetically together.

Concepts

absolute value

addition- addend
- indicated addition

algebra- algebraic expression

altitude

amount

angle- acute
- adjacent
- arm
- complement
- complementary
- corresponding
- degree
- elevation
- exterior
- external
- included, contained
- internal
- obtuse
- right
- straight
- supplement
- supplementary
- vertex
- vertically opposite

annual

approximation- approximately

area

assumption

average

axiom

axis- horizontal
- vertical

bisector- right

check

circle- arc
- center
- concentric
- chord
- circumscribe
- circumference
- diameter
- inscribe
- point of contact
- quadrant
- radius
- segment
- sector
- semicircle
- tangent

coefficient- literal
- numerical

collect (terms)

combine (terms)

consecutive

corresponding points

corollary

cube root

decimal- point
- coefficient

definitions

degree- first
- second

diagonal

digit

dimensions

discount

division- dividend
- divisor
- quotient
- remainder

due

error- percentage
- relative

equation- equality
- identity
- left side
- linear
- members of
- right side
- root
- satisfy
- simple

Concepts (cont'd.)

- solution
- solve
- unknown
-

equidistant

evaluation - evaluate

expression- binomial
- compound
- monomial
- polynomial
- term of
- trinomial
- simple

factor- literal
- numerical

figure- irregular
- plane
- rectangular
- regular

fluctuation

formal proof

formula

fraction- common
- denominator
- numerator
- terms

fret

gallon

geometry- geometric

graph- axis
- bar
- broken line
- circle
- curved line
- motion
- pictorial

graphic representation

Hypotenuse Rule

inspection

integer- consecutive
- even
- odd

interest- rate

inverse

is equal to

length

line- broken
- curved
- horizontal
- segment of
- straight
- vertical

linear

liter

locus

margin of profit

mathematical relation

measurement- acre
- unit of

multiplication- multiplicand
- multiplier

negative

notation

number- algebraic
- arithmetic
- directed
- integer
- literal
- negative
- missing
- positive
- signed
- whole

numerical value

order- ascending
- descending

overhead

parentheses- brackets

parallel lines

percent

Concepts (cont'd.)

perimeter

perpendicular

pi

plane

point

positive

possible error

probable error

power- base
- exponent

principal

prism

product

pfofit

proposition- conclusion
- construction
- cnnverse
- enunciation
- hypotheses
- problem
- proof
- theorem

quantity

ratio

rectilinear figure-
- equiangular
- equilateral
- parallelogram
- polygon
- quadrilateral
- rectangle
- regular
- rhombus
- square
- trapezium

rule of grouping

rule of order

rule of transposition

scale

scale-drawing

signs- of operation
- of equality

simplify

solid- cone
- cube
- cylinder
- prism
- pyramid
- rectangular
- sphere

square

square root- root
- root sign
- radical sign

statistical material

substitution table

subtraction- indicated
- minuend
- subtract
- subtrahend

successive integral values

surface- curved
- plane

symmetry- axis of
- center of
- line
- plane
- point

terms- like, similar
- unlike, dissimilar

table of values

tabular form

thermograph

trasnposed

triangle- acute angled
- obtuse angled
- right angled
- base

Concepts (cont'd.)

triangle- congruent
- corresponding angles of
- equiangular
- equilateral
- hypotenuse
- isosceles
- median
- scalene
- sides
- similar
- vertex
- vertices

unknown quantity

verify

width

C Analysis of the text for Routines or Techniques

The initial attempt at classifying the various routines was in a manner similar to that used in classifying the relationships- i.e., a listing of routines under various main headings. This resulted in much duplication of routines under various headings, and for this reason it was decided to develop this analysis by a chronological listing of the routines involved in each individual chapter. It is often difficult to differentiate between techniques and relationships, and in several instances it might be debated whether a particular routine would best be classified under relationships. For this reason it is necessary that one keeps in mind that there is no definite border line between these various categories, and all classifications made are based on the personal opinion of the writer and his "working definitions" of these three important components of the grade IX mathematics course.

The following 125 techniques or routines approximate closely their order of appearance in the various chapters concerned, and do not under any circumstances, represent the order in which they should be taught in the class. It must also be remembered that this is only a listing of the routines found in the text-book, and not a full description of the routines themselves.

PART I--ALGEBRA FOR TODAY

Routines found in Chapter 1 " How Letters are Used in Algebra"

1. Using letters to represent various concepts, as length and width.

2. Combining similar terms to find their sums or differences.

3. Evaluating formulas and simple algebraic expressions

4. Simplifying formulas involving dissimilar terms.

5. Finding sums, differences, products and quotients in Algebra.

6. Using formulas in mensuration:

a. finding the area of a rectangle
b. finding the volume of a rectangular solid
c. finding the area of a triangle
d. finding the volume of a prism
e. finding the area of a square
f. finding the volume of a cube
g. finding the circumference of a circle
h. finding the area of a circle
i. finding the volume of a cylinder.

7. Using formulas in business:

a. finding interest by formula
b. finding amount by formula
c. finding profit by formula

8. Using formulas in laboratories and shops

9. Expressing relationships by:

a. a formula
b. a table
c. a graph

10. Obtaining formulas from verbal statements

11. Obtaining verbal statements from formulas.

Routines found in Chapter 2 "The Equation"

1. Solving problems by the equation method

2. Finding the missing number in an equation

3. Finding the missing value in a table

4. Checking results by substituting the value found for the missing number in the given equation

5. Finding the root of an equation (finding the solution of the equation).

Routines found in Chapter 3 "Problems"

1. Solving simple problems involving number relations.

2. Solving simple geometric problems.

3. Solving business problems.

4. Solving miscellaneous problems.

5. Application of the 5 recommended steps in problem solving.

Routines found in Chapter 4 "Graphs"

1. Making statistical graphs as bar graphs, etc.

2. Interpreting statistical graphs

3. Graphing a formula

4. Interpreting the graph of a formula

5. Solving problems graphically.

Routines found in Chapter 5 "Signed Numbers"

1. Representing fluctuations by signed numbers

2. Picturing signed numbers

3. Adding signed numbers

4. Subtracting signed numbers

5. Multiplying signed numbers

6. Dividing signed numbers

Routines found in Chapter 6 "Fundamental Operations"

1. Addition of monomials
2. Subtraction of monomials
3. Addition of polhnomials
4. Subtraction of polynomials
5. Rule of signs for multiplication
6. Rule of order for multiplication
7. Rule of grouping for multiplication
8. Rule of exponents for multiplication
9. Multiplication of monomials
10. Order of operations of algebraic expressions of any power
11. Multiplication of a polynomial by a polynomial
12. Multiplication of a polynomial by a monomial
13. Rule of signs for division
14. Rule of exponents for division
15. Division of a monomial
16. Division of a polynomial by a monomial
17. Division of a polynomial by a polynomial
18. Double use of the plus and minus signs
19. Checking in order to make sure that the result obtained is correct, by substituting for each letter appearing in the given expression, some convenient numerical value.
20. Application of the rule for removing parentheses.

Routines found in Chapter 7 "Equations of the First Degree in One Unknown.

1. Solving an equation by interchanging members
2. Transforming an equation by addition and subtraction
3. Solving equations with negative roots
4. Changing signs in an equation by multiplying by -1
5. Solving an equation involving parentheses
6. Solving equtions with fractional coefficients
7. Solving literal equations
8. Solving percentage probdems
9. Solving motion problems
10. Solving mixture problems
11. Solving miscellaneous problems.

PART II-- GEOMETRY FOR TODAY

Routines found in Chapter 8 "First Steps in Geometry"

1. Measuring lines and angles by rule and protractor
2. Drawing to scale
3. Finding areas of:
   a. rectangles
   b. parallelograms
   c. triangles
   d. circles
   e. other plane figures
4. Bisecting a given line (Construction 1)
5. Bisecting a given angle (Construction 2)
6. Constructing a perpendicular to a given line at a point in the line (Construction 3)
7. Constructing a perpendicular to a given line from a given point outside of the line (Cons. 4)
8. Naming triangles by their angles and their sides

9. Constructing an angle equal to a given angle (Construction 5)

10. Constructing congruent triangles.

Routines found in Chapter 9 "Angles"

1. Interpreting the meaning of theorems

2. Differentiating between complementary, supplementary, and vertically opposite angles

3. Determining the sum of the angles at a point.

Routines found in Chapter 10 "Parallel Straight Lines"

1. Differentiating parallel straight lines from non-parallel straight lines.

2. Testing converses

3. Through a given point to construct a straight line parallel to a given straight line (Construction 6).

Routines found in Chapter 11 "Angles of a Triangle; Formal Proofs"

1. Techniques on 'How to Prove Theorems and Solve Problems'

2. Dividing angles of plane figures

3. Calculating the sum of interior and exterior angles of a polygon.

Routines found in Chapter 12 "Congruence"

1. Constructing congruent triangles

2. Measuring distances indirectly by using the concept of congruence

3. Naming corresponding parts of congruent triangles

4. Using a congruence theorem to verify the method

of bisecting a given straight line (Construction 1)

5. Using a congruence theorem to verify the method of bisecting a given angle (Construction 2)

Routines found in Chapter 13 "Practice on Essentials"

1. Recognizing and selecting the particular congruence theorem which fits a given situation
2. Drawing accurate figures for given exercises.

Routines found in Chapter 14 "Parallelograms"

1. Distinguishing various types of quadrilaterals
2. Expressing a problem either with or without respect to a particular lettered figure.

Routines found in Chapter 15 "Similarity and Symmetry"

1. Constructing similar figures by double compasses and by pantograph.
2. Application of concept of similarity to mensuration problems.
3. Using symmetry in everyday life.

Routines found in Chapter 16 "Constructions"

1 Planning solutions in solving exercises
2. Using additional aids in solving exercises in geometry
3. Dividing a line into equal parts (Const. 7)
4. Using the Diagonal Scale to measure lengths to hundredths of an inch.

Routines found in Chapter 17 "Areas"

1. Application of theorems 20,21, and 22 to areas of parallelograms and triangles
2. Constructing a triangle equal in area to a given quadrilateral (Construction 8)

Routines found in Chapter 18 "Loci"

1. Finding the locus of points at a fixed distance from one given point
2. Finding the locus of points at a fixed distance from two given points
3. Finding the locus of points at a fixed distance from three given points
4. Finding the locus of points equidistant from two intersecting straight lines.

Routines found in Chapter 19 "Theorem of Pythagoras"

1. Constructing a right angled triangle by applying the Hypotenuse Rule alone.
2. Finding the value of any side of a right angled triangle when the values of the other two sides are known.
3. Finding the side of a square when the area of the square is known (the proces of finding the square root)
4. Finding square roots from a table
5. Using a table of squares and square roots in applying the formula $c^2 = a^2 + b^2$, and in solving problems based on the Hypotenuse Rule.

CHAPTER VI

ANALYSIS OF THE EXAMINATION PAPER

The analysis of the examination paper though conducted in a manner similar to that used in chapter V, will be listed in a different format. In chapter V, the relationships found in the text were alphabetically arranged under 12 main headings; the 303 concepts were alphabetically listed under main topics; and the techniques or routines were chronologically listed under chapter headings of the book in which they were found.

In chapter VI the relationships and routines will be shown separately as they appear in each of the 53 questions on the examination paper. Because there is such a duplication of concepts on the various questions, the concepts will therefore be listed alphabetically with no attempt to group them under main headings, as was done in chapter V.

In some instances both the relationship and the routine seemed of equal importance and therefore each was listed under its separate heading. In other questions the relationship alone seemed the more important and in that case only the relationship was listed. Still in other questions, the routine itself seemed the only important factor, in which case only a routine was classified under that particular question.

A. Relationships found on the Examination Paper

The following are the relationships found in each of the 53 questions on the examination paper.

CHAPTER VI

ANALYSIS OF THE EVALUATING SHEET

The analysis of the evaluating sheet roughly conducted in a manner similar to that used in chapter V, will be listed in a different manner. In chapter V, the relationships found in the case were alphabetically arranged under 12 main headings; and the material was alphabetically listed under each heading; and all the routines or routines were chronologically listed under similar headings of the week in which they were found.

In chapter VI the relationships and routines will be given separately as they appear in each of the 55 questions on the evaluating sheet. Because there is much duplication of comments in the various questions, this chapter will therefore be in a way supplementary with an attempt to group them under main headings, as was done in chapter V.

In some instances only the relationship and the routine seemed of equal importance and therefore both are listed under its separate heading. In other questions the relationship alone seemed the more important and in that case only the relationship was listed. Still in other questions, the routine itself seemed the only important factor, in which case only a routine was placed under that particular question.

1. Relationships Found in the Evaluating Sheet

The following are the relationships found in each of the 55 questions on the evaluating sheet.

Question 1

No major relationship exists

Question 2

1. If the sum of two angles is equal to 90 degrees the two angles are called complementary.

Question 3

No major relationship exists

Question 4

1. The area of a rectangular surface is equal to the product of its length and its width.

Question 5

1. Expressing a relationship between two magnitudes in equation form.

Question 6

1. The area of a circle is equal to its radius squared times "pi".

Question 7

1. The relationship between two fractions is best expressed as a ratio.

Question 8

No major relationship exists

Question 9

1. The area of a rectangular surface is a function of its length and its width.

Question 10

1. The size of an angle subtended by an arc of a semicircle is a function of the arc of that semicircle.

Question 11

1. Expression of the relationship of equality between two magnitudes.

Question 12

1. The sum of the angles of a straight angle is equal to 180 degrees.

Question 13

No major relationship exists

Question 14

1. The sum of the interior angles of a polygon is proportional to the number of sides of the polygon.

2. The sum of the angles of a triangle is equal to 180 degrees.

Question 15

1. When two figures are similar, the ratio of any two corresponding sides equals the ratio of any other pair of corresponding sides.

Question 16

No major relationship exists

Question 17

No major relationship exists

Question 18

1. If two triangles are congruent their corresponding sides must be equal, and their corresponding angles must also be equal.

2. If two triangles have 2 sides and the included angle of one respectively equal to 2 sides and the included angle of the other, the triangles are congruent.

3. If two triangles have two angles and a side of one respectively equal to two angles and the corresponding side of the other, the triangles are congruent.

Question 19

No major relationship exists

Question 20

1. The relation of "inches per second" to "feet per minute" may be expressed as a ratio.

Question 21

1. The converse of a theorem is found by interchanging the conclusion and the given part of the theorem.

Question 22

1. The volume of a cylinder is proportional to the product of its height and the area of its base.

Question 23

1. When a straight line cuts two parallel straight lines, the corresponding angles are equal. (Theorem 8)

2. The sum of the angles of a straight angle is equal to 180 degrees.

Question 24

No major relationship exists

Question 25

No major relationship exists

Question 26

1. When two figures are similar:

   a. their corresponding angles are equal

   b. the ratio of any two corresponding sides equals the ratio of any other pair of corresponding sides.

2. In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

Question 27

No major relationship exists

Question 28

1. Expression of a relationship of equality between two magnitudes.

Question 29

1. The locus of a point at a fixed distance from a given point is a circle.

Question 30

1. The relationship between two numbers, whole or fractional may be expressed as a ratio or fraction.

Question 31

No major relationship exists

Question 32

1. The volume of a cylinder is proportional to the product of the area of its base multiplied by its height.

Question 33

1. In a right angled triangle the square of the hypotenuse is equal to the sum of the squares on the other two sides.

2. The opposite sides of a rectangle are equal.

Question 34

1. The volume of a cube is the product of its length, width, and its height.

2. The length, width, and height of a cube are equal.

Question 35

No major relationship exists

Question 36

1. The relationship between two ratios may be shown by expressing one fraction as a % of the other fraction.

Questinn 37

1. The area of a sector of a circle varies directly as its radius squared and the angle subtended by the arc of the sector.

Question 38

1. Expression of a relationship of equality between two magnitudes.

Question 39

1. In a right angled triangle the square of the hypotenuse equals the sum of the squares on the other two sides.

2. The area of a parallelogram equals the product of its base and its height.

Question 40

1. Parallelograms on the same base and between the same parallels are equal in area. (Theorem 20)

2. Triangles on the same base and between the same parallels are equal in area. (Theorem 21)

3. If a triangle and a parallelogram stand on the same base and between the same parallels the area of the triangle is half that of the parallelogram. (Theorem 22)

4. The area of a triangle is equal to half the product of its base and its height.

5. The opposite sides of a parallelogram are equal. (Theorem 17)

Questions 41-50 inclusive

Questions 41-50 inclusive are mainly questions involving routines and no major relationships exist.

Question 51

1. Triangles on the same base and between the same parallels are equal in area. (Theorem 21)

Question 52

1. The sum of the angles of a triangle is equal to 180 degrees.

2. If the corresponding angles formed by the intersection of a transversal with two straight lines are equal, then the two straight lines are parallel.

3. If two straight lines intersect, the vertically opposite angles are equal. (Theorem 5)

4. If two equal angles add up to 180 degrees, each must be a right angle.

Question 53 a,b,c,d.

1. A graphical representation of the relationship between Canada's oil production in 1938 with that of 1937.

2. A graphical representation of the relationship between Alberta's oil production in 1943 with that of 1942.

3. A graphical representation of Canada's total oil production during the period 1938-1941compared with the total production in Canada for the period 1942-45.

4. Expression of the relationship of the 13 year period, 1933-45 to Alberta's average annual oil production in bbls.

B. Concepts found on the Examination Paper

The following list of 99 concepts, alphabetically arranged comprises the technical vocabulary found in the grade IX examination paper. The numbers in brackets after each concept is the number of times that that particular term appeared on the examination paper.

angle (5)-complement (1)
-corresponding (1)
-interior (1)
-right (1)
-vertically opposite (1)

annual (1)

approximate (4)

area (20)

average (2)

bbls. (2)

base (5)

bears (2)

bisector (1)

brackets (1)

circle (3)

collect (1)

compare (1)

congruent (1)

converse (1)

cube (2)

cubic inch (6)

cut (1)

cylinder (2)

degree (2)

diameter (2)

diamensions (1)

directly (1)

divide (1)

divided by (1)

equal (5)

equation (6)

equivalent to (1)

exceed (2)

factor (2)

feet per minute (1)

given figure (4)

find the value of (1)

fractional number (1)

graph (1)

greatest value (1)

height (8)

inches per second (1)

is equal to (5)

length (6)

locus (1)

multiply (5)

multiplied by (4)

nearest (1)

number (1)

pair (1)

Concepts (cont'd.)

parallel (//) (3)

parallel lines (1)

parallels (1)

parallelogram (//m) (6)

pentagon (1)

percent (%) (2)

percentage (2)

perimeter (2)

perpendicular (3)

pi (π) (11)

plane (2)

points (5)

produtt (6)

production (9)

propositions (1)

quotient (2)

radius (3)

ratio (6)

rectangle (2)

remainder (1)

satisfies (1)

sector (1)

semicircle (1)

sides (1)- corresponding (1)

simplify (1)

square (2)

square-inch (17)

square root ($\sqrt{\ }$) (1)

statement (3)

straight line (2)

subtract (1)

sum (2)

transversal (1)

triangle (9)-congruent (1)
-isosceles (1)
-right-angled (1)
-similar (1)

unit (1)

value (1)

varies directly (1)

volume (3)

width (3)

whole (1)-number (1)

Of the 99 concepts used in the examination, all but one are found in the text. This one term is "bears" which appears twice on the examination.

Remembering that 303 separate concepts were listed in chapter V as comprising the technical vocabulary of the text, it is now possible to calculate the percentage of text-book coverage made by the examination paper.

$$\% \text{ coverage} = \frac{99}{303} \times 100$$

$$= 32.7$$

The percentage of text-book coverage made by the examination paper is 32.7- i.e., almost 1/3 of all the concepts found in the text-book were also found one or more times on the examination paper.

C. <u>Routines found on the Examination Paper</u>

The following are the routines found in the 53 examination questions on the exam. paper.

<u>Question 1</u>

1. Finding the product of two decimal fractions

<u>Question 2</u>

1. Finding the complement of an angle

<u>Question 3</u>

1. Calculating the lowest common denominator and then finding the sum of three fractions.

<u>Question 4</u>

No routines involved

<u>Question 5</u>

1. Solving for the unknown "N" and calculating the product of 16 and (N/2).

Question 6

No routines involved

Question 7

1. Finding the lowest common denominator and equating the ratios.

Question 8

1. Finding the average of a set of numbers by summing the numbers and dividing the sum by their number (5).

Question 9

No routines involved

Question 10

1. Calculating the angles at 0 subtended by equal arcs of a semicircle.

Question 11

1. Application of the rule for removing parentheses
2. Collecting like terms.
3. Solving for the value of the unknown "a".

Question 12

No routines involved

Question 13

1. Finding the sum of a group of signed numbers.

Question 14

1. Finding the sum of the interior angles of a pentagon by calculating the sum of the angles of the smallest number of triangles making up the polygon.

Question 15

1. Finding with ratio, the length of a side of a triangle that is similar to another given triangle.

Question 16

1. Application of the rule of exponents for multiplying

2. Application of the rule of signs for multiplying.

3. Multiplying a monomial by a monomial.

Question 17

1. Testing the equality of equations by reducing to the lowest common denominator.

Question 18

1. Finding the third angle of a triangle when the other two angles are known.

2. Application of the congruence theorems to given triangles.

Question 19

1. Application of the rule of signs for multiplying.

2. Application of the rule of exponents for multiplication.

3. Finding the product of a set of numbers.

Question 20

1. Equating "feet per minute" to "inches per second".

Question 21

1. Testing converses

Question 22

1. Calculating the volume of a cylinder of given diameter and height.

Question 23

1. Application of the interrelationship of corresponding angles.

Question 24

1. Finding the numerical value of a given algebraic expression by substituting known values in the given formula.

Question 25

1. Application of the four fundamental operations to a set of given fractions in order to find the algebraic expression having the greatest value.

Question 26

1. Application of the rule of similar triangles to find the equal angles.

2. Applying the rule of similar triangles to find which sides of given triangles are in proper ratio.

3. Application of the Hypotenuse Rule to find the third side of a right angled triangle when the other two sides are known.

Question 27

1. Application of the rule of exponents for dividing.

2. Division of a monomial by a monomial.

Question 28

1. Application of the rule for removing parentheses

2. Transposing and collecting like terms

3. Solving for the value of the unknown "n".

Question 29

1. Finding the locus of all points on a plane that are a given distance from two given poins on the same plane.

Question 30

1. Finding a number that bears the same ratio to a given number as 3 bears to 8.

Question 31

1. Application of the rule of signs for division

2. Application of the rule of exponents for division

3. Dividing a polynomial by a monomial.

Question 32

No routines involved.

Question 33

1. Calculating the third side of a right angled triangle with the Hypotenuse Rule.

2. Application of Theorem 17.

Question 34

1. Finding the cube root of 512 by trial and error.

Question 35

1. Calculating a given percentage of a given number.

Question 36

1. Finding the difference between two fractions by use of L.C.D.

2. Finding the ratio of a given fraction to another given fraction.

Question 37

No routines involved

Question 38

1. Multiplying through by 12 to clear fractions.

2. Application of the rule for removing parentheses.

3. Transposing and collecting like terms

4. Solving the equation for "y".

Question 39

1. Finding the third side of a right angled triangle by the Hypotenuse Rule.

2. Application of Theorem 17 to find the length of a side of a parallelogram.

3. Finding the area of the parallelogram ABCD by multiplying its base by its height.

Question 40

1. Application of Theorem 20
2. " " " " " 21
3. " " " " " 22
4. " " " " " 17
5. Application of the relationship that the area of a triangle is equal to the product of its base times its altitude, divided by two.

Question 41

1. Finding the L.C.D.
2. Combining similar terms

Question 42

1. Division of a decimal fraction by a decimal fraction.

Question 43

1. Finding the product of a mixed number and a common fraction.

Question 44

1. Division of a common fraction by another common fraction.

Question 45

1. Application of the rule of order for arranging terms.
2. Application of the rule of exponents for subtraction.
3. Subracting a polynomial from a polynomial
4. Application of the rule of subtraction for signed numbers.

Question 46

1. Application of the rule of order of terms
2. Applcation of the rule of exponents for division
3. Application of the rule of signs for division.
4. Applying rule of subtraction for signed numbers.

Question 47

1. Application of the rule of exponents for addition.

2. Adding a polynomial to a polynomial.

3. Applying the rule of exponents for division.

4. Applying the rule of signs for division.

5. Dividing a polynomial by a polynomial.

Question 48

1. Application of the rule for removing parentheses

2. Applying the rule of order of operations in solving algebraic expressions.

3. Applying the rule of signs

4. Collecting like terms

5. Solving for the unknown value "a".

Question 49

1. Applying the rule of signs for division

2. Applying the rule of exponents for division

3. Division of a monomial by a monomial.

Question 50

1. Transposing like terms to one side of the equation.

2. Finding the L.C.D.

3. Collecting like terms.

4. Solving for the unknown value "K".

Question 51

1. Drawing the line DE parallel to BC according to directions.

2. Joining the points DC and BE to form the required triangles ABE and ACD

3. Applying Theorem 21

4. Applying "How to Prove Theorems and Solve Problems" as outlined in chapter XI of the text.

Question 52

1. Applying "How to Prove Theorems and Solve Problems" to a particular problem.

2. Drawing the figure according to the stated problem.

3. Application of the 4 relationships listed under question 52, Part A, page 116 to the particular problem.

Question 53a

1. Graphical interpretation to find Canada's oil production in certain years.

2. Finding the difference in production between various years, by correct interpretation of the units on the graph.

3. Expressing this obtained difference as a percentage of 1937 oil production.

Question 53b

1. Graphical interpretation to find Alberta's oil production for certain years.

2. Finding the difference in production between various years by correct interpretation of the graph units.

3. Expressing this obtained difference as a percentage of the oil production of a certain year.

Question 53c

1. Interpreting from the graph the oil production in Canada for two 4 year periods.

2. Comparing these production figures as an approximate ratio

Question 53d

1. Interpreting from the graph, the annual oil production in Alberta over a period of time.

2. Finding the average annual oil production by summing the production for each of the years from 1933-45 and dividing by the number of years concerned, namely 13.

CHAPTER VII

PUPIL ACHIEVEMENT ON RELATIONSHIPS, CONCEPTS, & ROUTINES

The development of chapter VII is based on evidence of what the pupils know rather than what they do not know. As mentioned previously, almost every question is actually a combination of relationships, concepts and routines, and it is often very difficult to determine whether errors made were due to faulty relational thinking, or to improper performance of routines. This is especially applicable to section A where all the questions are of a multiple choice nature, and where also the element of guessing has very likely played an active part.

In calculating pupil achievement, each question was considered separately as to "correct relationships, and/or concepts, and/or routines". With reference to the first 40 questions, if full marks depended mainly on one of these 3 classifications, then that classification was given a score value of 2 marks (full marks for the question). If full marks depnded on a combination of two or all of the classifications, the two major ones were each given a score value of one mark, and the third classification, (usually concepts) though important, was given a zero value. It must be remembered that in section A no partial marks were given- the students received either zero marks or two marks. It is this method of marking, plus the type of question (multiple choice) which makes it extremely difficult to say whether errors made were due to faulty relationships, concepts, or routines.

It is much easier, yet more correct, to calculate the number of students receiving full marks on each question in section A and say, "these students know the following particular relationships, concepts, and routines, than it is to attempt to calculate the number of students that do NOT know the various relationships, concepts, and routines involved in the 40 questions of section A.

In section B the first 10 questions involve only routines and concepts, and an analysis of these questions allows one to more accurately determine what errors were made in the various routines.

In section A and B the number and percent of students receiving full marks (and thus having all the correct relationships, routines and concepts involved) were calculated, and besides these was listed the value alloted (VA) for each particular relationship, routine or concept. This allotment of marks was arbitrarily made by the writer, for the sole purpose of calculating pupil achievement, and does not represent the method of marking as used by the examination board, necessarily. Whenever possible, the errors made were classified as either incorrect relationships, concepts or routines; but if it were not possible to differentiate the reason for the error, the errors were then grouped together, as for example, 'Incorrect relationship and/or routine'.

In section B, questions 41-50 inclusive, the errors were classified under one or more of the routines involved as listed in chapter VI, part C. In questions 51 and 52, the number and percent of students showing the correct relationship and routine are listed as well as the

number and percent of students making errors on the particular relationships and routines involved in the problem.

In the following pages three symbols will be used consistently, with the following meanings:

N-- the number of students

%-- the number of students expressed as a % of 300.

VA-- the value alloted (by the writer) for the particular classification under discussion.

Question	N	%	VA
<u>Question 1</u>			
Correct Routine	164	55	2
Correct Concept	164	55	0
Incorrect Routine	134	45	
<u>Question 2</u>			
Correct Relationships	212	71	1
Correct Concepts	212	71	0
Correct Routines	212	71	1
Incorrect Relationship	74	25	
Incorrect Routine	13	4	
<u>Question 3</u>			
Corsect Routine	261	87	2
Incorrect Routines	37	12	
<u>Question 4</u>			
Correct Relationships	22	7	1
Correct Concepts	22	7	1
Incorrect Relationships	174	58	
Incorrect Concepts	104	35	
<u>Question 5</u>			
Correct Routine	184	62	1

	N	%	VA
Correct Relationship	184	62	1
Incorrect Routine	10	3	
Incorrect Relationship	96	32	
Question 6			
Correct Relationships	56	19	1
Correct Concepts	56	19	1
Incorrect Relationships	146	49	
Incorrect Concepts	87	29	
Question 7			
Correct Concepts	129	43	1
Correct Routine	129	43	1
Incorrect Concepts	144	48	
Incorrect Routine or Guessing	20	7	
Question 8			
Correct Relationship	233	78	1
Correct Routine	233	78	1
Correct Concept	233	78	0
Incorrect Relationship	21	7	
Incorrect Routine	40	13	
Guessing	5	2	
Question 9			
Correct Relationship	91	30	2
Correct Routine	91	30	0
Incorrect Relationship	187	62	
Incorrect Routine or Guessing	13	4	
Question 10			
Correct Relationship	264	88	1
Correct Routine	264	88	1
Correct Concepts	264	88	0

	N	%	VA
Incorrect Relationship	13	4	
Incorrect Concept	21	7	
<u>Question 11</u>			
Correct Relationship	122	41	0
Correct Routine	122	41	2
Correct Concept	122	41	0
Incorrect Routine	157	52	
Incorrect Concept	21	7	
<u>Question 12</u>			
Correct Relationship	232	77	1
Correct Concepts	232	77	1
Incorrect Concepts	67	23	
<u>Question 13</u>			
Correct Routine	242	81	2
Correct Concept	242	81	0
Incorrect Routine	54	18	
<u>Question 14</u>			
Correct Relationship	99	33	1
Correct Routine	99	33	1
Correct Concepts	99	33	0
Incorrect Routines	201	67	
<u>Question 15</u>			
Correct Relationship	51	17	1
Correct Routine	51	17	1
Incorrect Routines	248	83	
<u>Question 16</u>			
Correct Routines	95	32	2
Incorrect Routines	205	68	

	N	%	VA
Question 17			
Correct Relationship	173	58	0
Correct Routine	173	58	2
Incorrect Routine	117	39	
Question 18			
Correct Relationship	121	40	1
Correct Routine	121	40	1
Correct Concepts	121	40	0
Incorrect Relationships	178	59	
Question 19			
Correct Routine	136	45	2
Correct Concept	136	45	0
Incorrect Routine	153	51	
Question 20			
Correct Relationship	84	28	1
Correct Routine	84	28	1
Correct Concept	84	28	0
Incorrect Relationship	156	52	
Incorrect Routine	53	18	
Question 21			
Correct Relationship	71	24	1
Correct Concept	71	24	1
Incorrect Relationships	227	76	
Question 22			
Correct Relationship	112	37	1
Correct Routine	112	37	1
Correct Concept	112	37	0
Incorrect Relationships	109	36	
Incorrect Routine	169	56	

	N	%	VA
<u>Question 23</u>			
Correct Relationship	138	46	1
Correct Routine	138	46	1
Correct Concepts	138	46	0
Incorrect application of Relations	162	54	
<u>Question 24</u>			
Correct Routine	107	36	2
Correct Concepts	107	36	0
Incorrect Routines	168	56	
<u>Question 25</u>			
Correct Routines	105	35	2
Incorrect Routines	195	63	
<u>Question 26</u>			
Correct Reltionships	13	4	1
Correct Routines	13	4	1
Correct Concepts	13	4	0
Incorrect Relationships	282	94	
<u>Question 27</u>			
Correct Relationships	92	31	0
Correct Routines	92	31	1
Correct Concepts	92	31	1
<u>Question 28</u>			
Correct Routines	195	65	2
Incorrect Routines	88	29	
<u>Question 29</u>			
Correct Relationships	81	27	1
Correct Routines	81	27	1
Correct Concepts	81	27	0
Incorrect Relationships	211	70	

	N	%	VA
Question 30			
Correct Relationships	36	12	1
Correct Routines	36	12	1
Correct Concepts	36	12	0
Incorrect Relationships	117	39	
Incorrect Routines	125	42	
Question 31			
Correct Concepts	125	42	1
Correct Routines	125	42	1
Incorrect Routines	155	52	
Question 32			
Correct Relationship	154	51	1
Correct Concept	154	51	1
Incorrect Relationship	142	47	
Question 33			
Correct Relationship	40	13	1
Correct Routine	40	13	1
Incorrect Relationships	248	83	
Question 34			
Correct Relationship	171	57	1
Correct Routine	171	57	1
Correct Concepts	171	57	0
Incorrect Routines or Relationships	108	36	
Question 35			
Correct Routine	117	39	2
Incorrect Routine	179	60	
Question 36			
Correct Relationship	41	14	0
Correct Routine	41	14	1

	N	%	VA
Correct Concept	41	14	1
Incorrect Routines	238	79	
<u>Question 37</u>			
Correct Relationship	120	40	1
Correct Concepts	120	40	1
Incorrect Relationships	104	35	
Correct Relations- (marked incorrect)	59	20	
<u>Question 38</u>			
Correct Routines	58	19	2
Incorrect Routines	221	74	
<u>Question 39</u>			
Correct Relationship	107	36	1
Correct Routines	107	36	1
Correct Concepts	107	36	0
Incorrect Relationships	170	53	
<u>Question 40</u>			
Correct Relationships	160	53	1
Correct Routines	160	53	1
Correct Concepts	160	53	0
Incorrect Relationships	131	44	
<u>Question 41</u>			
Correct Routines	127	42	1
Incorrect Routines	167	66	
<u>Question 42</u>			
Correct Routines	96	32	1
Incorrect Routines	204	65	
<u>Question 43</u>			
Correct Routine	186	62	1
Incorrect Routines	109	36	

	N	%	VA
<u>Question 44</u>			
Correct Routines	226	75	1
Incorrect Routines	67	32	
<u>Question 45</u>			
Correct Routines	67	32	2
Incorrect Routines:	58		
a. Arranging terms	58	19	
b. Rule of exponents	29	10	
c. Rule of signs	38	13	
d. Terms added,not subtracted	51	17	
e. Mech. error in addition and subtraction	35	12	
f. Miscellaneous	15	5	
<u>Question 46</u>			
Correct Routines	58	19	2
Incorrect Routines:			
a. Arranging terms	30	10	
b. Rule of exponents	20	7	
c. Rule of signs for division	26	9	
d. Mech. error in division	73	24	
e. Miscellaneous errors	16	5	
<u>Question 47</u>			
Correct Routine	83	28	3
Incorrect Routines:			
a. Rule of exponents	40	13	
b. Mech. errors in addition and subtraction	25	8	
c. Mech. errors in division and multiplication	50	17	
d. Rule of signs	26	9	
e. Miscellaneous errors	16	5	
<u>Question 48</u>			
Correct Routines	76	25	2
Incorrect Routines:			
a. Rule for removing brackets	55	18	
b. Rule of signs	34	11	
c. Collecting like terms	19	6	
d. Miscellaneous errors	19	6	
e. Mechanical errors	65	32	

	N	%	VA
Question 49			
Correct Routines	110	37	2
Incorrect Routines:			
a. Rule of signs	5	2	
b. Rule of exponents	78	26	
c. Mech. errors in division	12	4	
d. Miscellaneous errors	62	21	
Question 50			
Correct Routines	55	18	4
Incorrect Routines:			
a. Transposing terms	55	18	
b. Error in L.C.D.	24	8	
c. Collecting terms	21	7	
d. Errors in signs	16	5	
e. Mech. error in subtraction	22	7	
f. Mech. error in multiplication	8	3	
g. Miscellaneous errors	18	6	
Question 51			
Correct Relationships:			
a. $\triangle DEB \cong \triangle DEC$	5	2	1
b. $\triangle ADE$ is a common triangle	2	1	1
c. $\triangle ABE \cong \triangle ACD$	1	0	1
Correct Routines:			
a. Join DC and BE	52	17	1
b. Apply Theorem 21	2	1	1
c. Apply Assumption 2	1	0	1
Incorrect Relationships :			
a. $\triangle DEB \cong \triangle DEC$	295	98	
b. $\triangle ADE$ is a common triangle	298	99	
c. $\triangle ABE \cong \triangle ACD$	299	100	
Incorrect Routines:			
a. Join DC and BE	210	70	
b. Application of Theorem 21	298	99	
c. Application of Assumpt. 2	298	99	
Question 52			
Correct Relationships	0	0	3
Correct Routines:			
a. Correct figure drawn	39	13	1
b. Application of Theorem 17B	1	0	1
c. Application of the relationships listed for this question in Chap. VI, part C.	0	0	1

	N	%	VA
<u>Question 53a</u>			
Correct Relationship	7	2	1
Correct Routine	7	2	1
Incorrect Routine	276	92	
<u>Question 53b</u>			
Correct Relationship	40	13	1
Correct Routine	40	13	1
Incorrect Routine	234	78	
<u>Question 53c</u>			
Correct Relationship	21	7	1
Correct Routine	21	7	1
Incorrect Routine	185	62	
<u>Question 53d</u>			
Correct Relationship	3	1	1
Correct Routine	3	1	1
Incorrect Routine	297	99	

TABLE LXXXIII

SUMMARY OF ACHIEVEMENT IN RELATIONSHIPS

Q	TVQ	VR	FS	RSO	TRSP	NA	MNA
2	2	1	212	212	300	1	1
4	2	1	22	22	300	0	0
5	2	1	184	184	300	10	10
6	2	1	56	56	300	11	11
8	2	1	233	233	300	1	1
9	2	2	91	182	600	1	2
10	2	1	264	264	300	2	2
12	2	1	232	232	300	1	1
14	2	1	99	99	300	0	0
15	2	1	51	51	300	1	1
18	2	1	121	121	300	1	1
20	2	1	84	84	300	7	7
21	2	1	71	71	300	2	2
22	2	1	112	112	300	19	19
23	2	1	138	138	300	0	0
26	2	1	13	13	300	5	5
29	2	1	81	81	300	8	8
30	2	1	36	36	300	22	22
32	2	1	154	154	300	4	4
33	2	1	40	40	300	12	12
34	2	1	171	171	300	21	21
37	2	1	120	120	300	17	17
39	2	1	107	107	300	23	23
40	2	1	160	160	300	9	9
51	6	3	1	3	900	38	114
		2	2	4			
		1	5	5			
52	6	3	0	0	900	71	213
53a	2	1	7	7	300	17	17
53b	2	1	40	40	300	26	26
53c	2	1	21	21	300	94	94
53d	2	1	3	3	300	75	75
	68	35	2931	3026	10500	499	718

KEY

Q: Question

TVQ: Total value of question

VR: Value alloted to relation.

FS: Frequency of scores

RSO: Raw score obtained on the particular relationship (RSO = VRxFS)

TRSP: Total raw score possible for all students on the particular relationship.

KEY

NA: No attempts

MNA: Total marks lost through students not attempting the particular question.

Total raw score possible for the 300 students on the above 30 questions involving relationships is 10,500 marks. The total raw score obtained by the 300 students on these 30 questions is 3026 marks.

Percentage achievement on relationships

$$= \frac{\text{RSO} \times 100}{\text{TRSP}}$$

$$= \frac{3026 \times 100}{10,500}$$

$$= 28.8$$

Total raw score possible for the 300 students on the 30 questions involving relationships is 10,500 marks. The total number of marks lost through students not attempting various questions involving relationships is 718 marks.

Percentage of marks lost through not attempting questions containing relationships $= \frac{\text{MNA} \times 100}{\text{TRSP}}$

$$= \frac{718 \times 100}{10500}$$

$$= 6.8$$

TABLE LXXXIV

SUMMARY OF ACHIEVEMENT IN CONCEPTS

Q	TVQ	VC	FS	RSO	TRSP	NA	MNA
4	2	1	22	22	300	0	0
6	2	1	56	56	300	11	11
7	2	1	129	129	300	7	7
12	2	1	232	232	300	11	.1
27	2	1	92	92	300	4	4
31	2	1	125	125	300	20	20
32	2	1	154	154	300	4	4
36	2	1	41	41	300	21	21
37	2	1	120	120	300	17	17
	18	9	971	971	2700	85	85

KEY

Q: Question

TVQ: Total value of question

VC: Value alloted to concept involved in question

FS: Frequency of scores receiving alloted value

RSO: Raw score obtained by all students on the particular concept. (RSO = VC x FS)

TRSP: Total raw score possible for all studemts on the particular concept.

NA: No attempts

MNA: Total marks lost by the students through not attempting the question containing the particular concept.

The summary of achievement in concepts is very narrow in scope in that only 9 questions are being used to calculate achievement. As mentioned previously, almost every question contains concepts to a greater or lesser extent, and it is difficult to apportion 2 marks among three categories. In the majority of cases, the

marks were divided between relationships and routines, and in the rare case where only relationships or routines existed, it was possible to allot marks for concepts. This allotment was done in the above 9 questions, all of them being found in section A of the examination paper. Because of the narrow scope of this summary of concepts, the results obtained must be considered only with reference to this inherent weakness.

The total raw score possible for all students on these 9 questions is 2700 marks. The total raw score obtained by the 300 students on these 9 questions is 971 marks.

Percentage achievement on concepts

$$= \frac{\text{RSO} \times 100}{\text{TRSP}}$$

$$= \frac{971 \times 100}{2700}$$

$$= 36$$

Total marks lost through not attempting the questions involving concepts is 85.

Percentage of marks lost through not attempting to answer the questions involving concepts

$$= \frac{\text{MNA} \times 100}{\text{TRSP}}$$

$$= \frac{85 \times 100}{2700}$$

$$= 3.2$$

TABLE LXXXV

SUMMARY OF ACHIEVEMENT IN ROUTINES

Q	TVQ	VR	FS	RSO	TRSP	NA	MNA
1	2	2	164	328	600	2	4
2	2	1	212	212	300	1	1
3	2	2	261	522	600	2	4
5	2	1	184	184	300	10	10
7	2	1	129	129	300	7	7
8	2	1	233	233	300	1	1
10	2	1	264	264	300	2	2
11	2	2	122	244	600	21	42
13	2	2	242	484	600	4	8
14	2	1	99	99	300	0	0
15	2	1	51	51	300	1	1
16	2	2	95	190	600	0	0
17	2	2	173	346	600	10	20
18	2	1	121	121	300	1	1
19	2	2	136	272	600	11	22
20	2	1	84	84	300	7	7
21	2	1	71	71	300	2	2
22	2	1	112	112	300	19	19
23	2	1	138	138	300	0	0
24	2	2	107	214	600	25	50
25	2	2	105	210	600	5	10
26	2	1	13	13	300	5	5
27	2	1	92	92	300	4	4
28	2	2	195	390	600	17	34
29	2	1	81	81	300	8	8
30	2	1	36	36	300	22	22
31	2	1	125	125	300	20	20
33	2	1	40	40	300	12	12
34	2	1	171	171	300	21	21
35	2	2	117	234	600	4	8
36	2	1	41	41	300	21	21
38	2	2	58	116	600	21	42
39	2	1	107	107	300	23	23
40	2	1	160	160	300	9	9
41	1	1	127	127	300	6	6
42	1	1	96	96	300	10	10
43	1	1	186	186	300	5	5
44	1	1	226	226	300	7	7
45	2	2	67	134	600	7	14
		1	1	1			
46	2	2	44	88	600	48	96
		1	58	58			
47	3	3	83	249	900	60	180
		1	37	37			
48	2	2	76	152	600	23	46
49	2	2	110	220	600	33	66
50	4	4	55	220	1200	81	324
51	6	3	1	3	900	38	114
		2	2	4			
		1	52	52			

TABLE LXXXV (cont'd.)

Q	TVQ	VR	FS	RSO	TRSP	NA	MNA
52	6	3	0	0	900	71	213
		2	1	2			
		1	39	39			
53a	2	1	7	7	300	17	17
53b	2	1	40	40	300	26	26
53c	2	1	21	21	300	94	94
53d	2	1	3	3	300	75	75
	107	75	---	8079	22500	919	1733

KEY

Q: Question

TVQ: Total value of question

VR: Value alloted to the routines involved

FS: Frequency of scores receiving value for routines

RSO: Raw score obtained on the particular routine by all students (RSO = VR x FS)

TRSP: Total raw score possible for all students on the particular routines

NA: No attempts

MNA: Total marks lost through not attempting the particular question involving routines.

Total raw score possible for the 300 students on the above listed 47 questions involving routines is 22,500 marks. The total raw score obtained by the 300 students on the above 47 questions is 8079 marks.

Percentage achievement on routines is $\frac{RSO \times 100}{TRSP}$

$$= \frac{8079 \times 100}{22500}$$

$$= 35.9$$

Total numbr of marks lost through students not attempting the questions involving the routines is 1733 marks.

Percentage of marks lost through no attempts on the routines is $\frac{MNA \times 100}{TRSP}$

$$= \frac{1733 \times 100}{22500}$$

$$= 7.7$$

D. Summary of Mechanical Errors on Questions 41-50

Because of the multiple choice questions in section A of the examination, it was impossible to accurately determine the number of mechanical errors made. Analysis of the answers of the first ten questinns in section B makes possible to a limited extent, a partial determination of mechanical errors.

The writer assumes that these ten questions, are representative samples of the whole paper, and conclusions reached through an analysis of these questioms can also be applied to the paper as a whole.

The data for columns F.AS and F.MD are taken from Chapter IV, "The Analysis of the Individual Answers on Each Test Item".

TABLE LXXXVI

SUMMARY OF MECHANICAL ERRORS

Q	TVQ	TAS	F.AS	M.AS	TMD	F?MD	M.MD
41	1	300	21	21			
42	1				300	33	33
43	1				300	41	41
44	1				300	34	34
45	2	600	35	70			
46	2	600	28	56	600	45	90
47	3	900	25	75	900	50	100
48	2	600	65	130			
49	2	600	17	34	600	12	24
50	4	1200	22	88	1200	8	32
	19	4800	213	474	4200	223	354

KEY

Q: Question

TVQ: Total value of question

TAS: Total raw score possible on question involving addition and subtraction.

F.AS: Frequency of mechanical errors in addition and subtraction.

M.AS: Total marks lost through errors in addition and subtraction.

TMD: Total raw score possible on questions involving multiplication and division.

F.MD: Frequency of mechanical errors in muliplication and division.

M.MD: Total marks lost through errors in multiplication and division.

It should be noted that under column M.MD for question 47, that the total marks lost through errors in multiplication and division is listed as only 100 instead of 150, even though M.MD is usually found by multiplying F.MD by TVQ. In question 47 those 50 students who made mechanical errors in multiplication and division all had correctly found the sum of $(P+Q)$, and therefore they were given one mark each for having a partially correct answer. Thus they lost only 2 marks each for the error in multiplication and division. Thus M.MD for question 47 would be (50 x 2), or 100.

% of marks lost through errors in addition and subtraction equals $\frac{M.AS \times 100}{TAS}$

$$= \frac{474 \times 100}{4800}$$

$$= 9.9$$

% of marks lost through errors in multiplication and division equals $\frac{M.MD \times 100}{TMD}$

$$= \frac{354 \times 100}{4200}$$

$$= 8.4$$

Adding the above calculated percentages we find that 18.3% of the marks lost on the first ten questions of section B were due to mechanical errors in the four fundamental operations. Generalizing, one might say that if 18.3% of the marks in section B were lost through

mechanical errors, it is very likely that the total percentage of marks lost on the whole examination paper due to mechanical errors would closely approximate 18% as well. It is astonishing and also inexcusable that pupils with B standing in mathematics should lose close to 20% of their marks on fundamental operations which they have been practicing for the last 9 school years.

CHAPTER VIII

PUPIL MASTERY OF THE COURSE ON THE BASIS OF TOPICS

The content of the examination paper has been divided into two main sections, Algebra and Geometry, and these in turn have been subdivided into individual main topics- 14 in the Algebra section and 11 in the Geometry section.

Classification of the examination questions under various topics was made in an arbitrary manner. Each question was analyzed into its various mechanisms and from such analysis the questions were classified under the various topics. Example: question 21 was listed under four different main topics as it was felt that this particular problem involved factors directly related to those four main topics. The net result of such analysis was the list of topics, alphabetically arranged, and enumerated below.

Algebra Topics

1. Averages
2. Equations, Equalities, and axioms
3. Evaluation and substitution
4. Exponents
5. Formulas
6. Fractions
7. Fundamental Operations applied to polynomials
8. Graphs
9. Mensuration (lengths, areas, volumes)
10. Parentheses
11. Percent
12. Problems
13. Ration and variation
14. Signed numbers

Geometry Topics

1. Angles
2. Congruence
3. Converses
4. Hypotenuse Rule
5. Loci
6. Measurement (lines, angles, areas)
7. Parallels
8. Parallelograms
9. Proofs (Problem Solving)
10. Similarity
11. Triangles

The following tables show the main topics, the questions containing the main topics, and pupil achievement based on the fully correct and partial scores for the questions involved under the various topics.

The following "key" will be used for the tables of pupil achievement on the algebra and geometry topics.

KEY

TRSP: Total raw score possible by all students on the questions containing the particular topic. This total is found by summing for all students, the raw scores total for the individual questions.

TRSO: Total raw score obtained by the 300 students on the questions containing the particular topic.

%-A: Percentage attainment. Found by the formula

$$\%\text{-A} = \frac{\text{TRSO} \times 100}{\text{TRSP}}$$

TPS: Total possible score for any one individual on the

particular topic. (The score referred to is the raw score value as found on the examination paper.)

MTS: Mean total score for the 300 students on the questions involved in the particular topic. This mean total score is found by taking the TPS (total possible score) and multiplying it by the "%-A" (percentage attainment)

An example will clarify the following tables. The first main topic under the algebra section is that of 'averages'. There were only two questions dealing with this particular topic, and they were 8 and 53 d, as seen in the second column of the table on algebra topics. Each of these questions was worth 2 marks, so the TRSP for the 300 students would be 300x2x2 which is 1200 marks. The column TRSO was computed by using the data of chapter IV, and for the topic of averages, the raw score total obtained by the 300 students was 472. TRSO was then expressed as a percent of TRSP which the table shows as 39. The total possible raw score for any one person to make on these two questions (8, 53d) was 4 marks as shown under TPS. The mean total score (MTS) 1.6 was found by taking 39% of the TPS, which means that the average score for the 300 students on the topic of averages is only 1.6 marks out of a possible total of 4 marks.

TABLE LXXXVII

ACHIEVEMENT ON ALGEBRA TOPICS

Topic	Questions	TRSP	TRSO	%-A	MTS	TPS
Averages	8,53d	1200	472	39	1.6	4
Equations	5,7,11,12, 17,20,23,26 28,32,34, 38,40,50.	9000	3865	43	12.9	30
Evaluation & Substitution	5,6,9,11, 20,22,24, 47.	5100	1798	35	6	17
Exponents	16,19,27, 31,45,46, 47,49.	5100	1639	32	5.4	17
Formulas	4,6,9,22, 32,34,39.	4200	1426	34	4.8	14
Fractions	1,3,7,17, 20,25, 30,35,36, 38,41,42, 43,44,50.	8400	3210	38	10.7	28
Fundamental Operations with Polynomials	11,16,28, 31,38,41, 45,46,47, 48,49,50.	7800	2451	31	8	26
Graphs	53a,b,c,d	2400	142	6	0.5	8
Mensuration	4,6,9,15, 22,24,34, 37,39.	5400	1674	31	5.6	18
Parentheses	5,11,13,16 19,28,38, 47,48.	5700	2502	44	8.4	19
Percent	35,36, 53a,53b.	2400	410	17	1.4	8
Problems (Easier problems)	3,8,13, 28,44.	2700	2088	77	6.9	9
Problems- Harder)	47,49,50 53a,d.	3900	765	20	2.6	13

TABLE LXXXVII- cont'd.

ACHIEVEMENT ON ALGEBRA TOPICS- cont'd.

Topic	Questions	TRSP	TRSO	%-A	MTS	TPS
Ratio and Variation	7,15,26, 30,36,37, 53c.	4200	822	20	2.8	14
Signed Numbers	13,16,19, 31,45,46, 47,48,49.	5700	2091	37	7	19

Using the data found in table LXXXVII, above it is now possible to calculate a mean percentage score-a weighted score that takes account of the variation in marks for the different topics.

The weighted mean percentage score is the sum of the products obtained by multiplying the percentage attainment (%-A) by the total possible raw score (TPS), for each of the main topics , and dividing this sum by the sum of the number of marks (raw scores) alloted for each main topic.

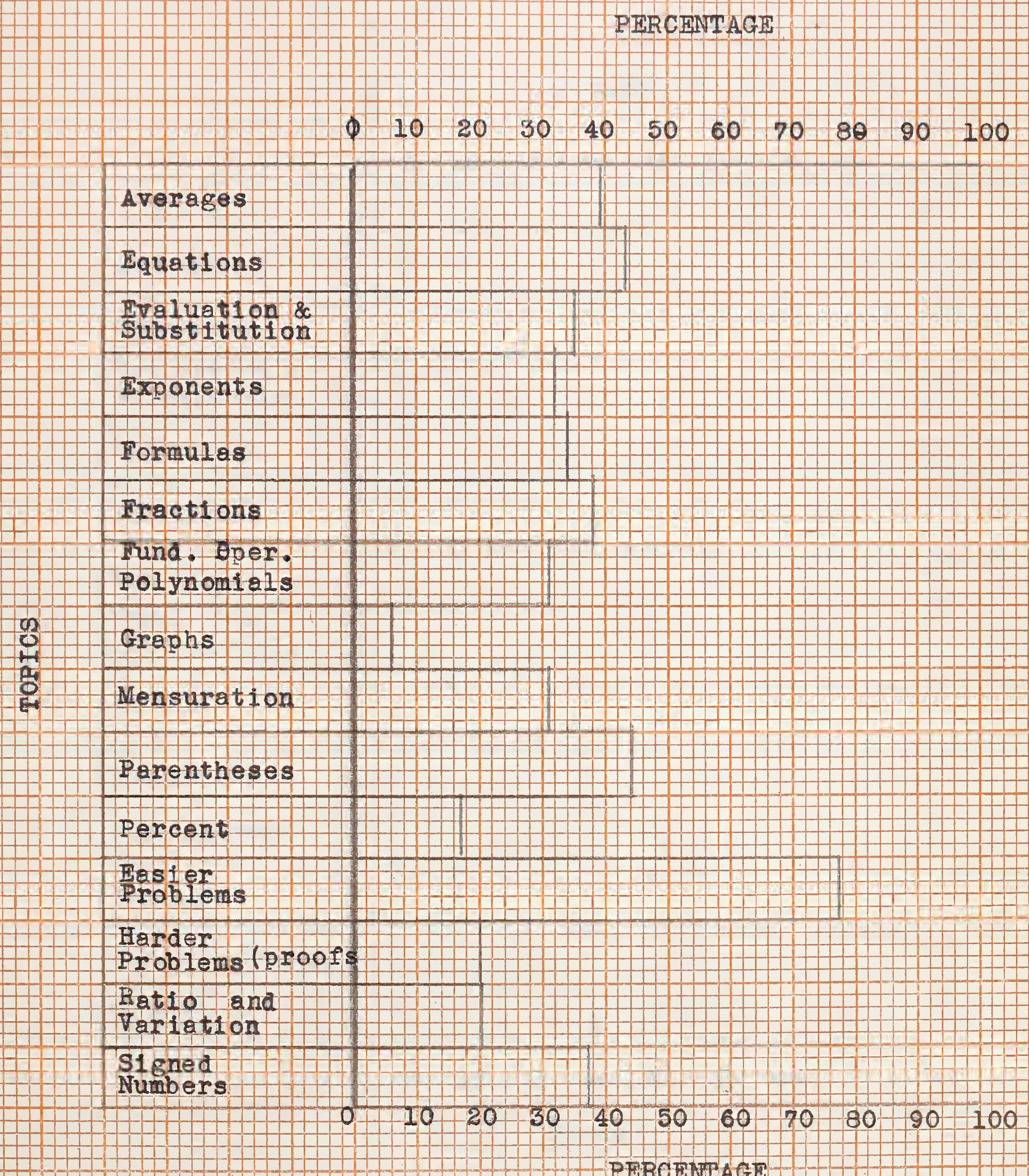
$$\text{Weighted Mean \% Score} = \frac{\sum(\%\text{-A})x(\text{TPS})}{\sum \text{TPS}}$$

$$= \frac{8035}{228}$$

$$= 35.2$$

PERCENTAGE OF "TOTAL RAW SCORE POSSIBLE" OBTAINED ON THE INDIVIDUAL TOPICS IN THE ALGEBRA SECTION OF THE EXAMINATION.

Figure 5.



The following graph shows the total raw score value for each of the main topics in the Algebra section of the examination paper.

Figure 6

TOTAL RAW SCORE VALUE PER TOPIC

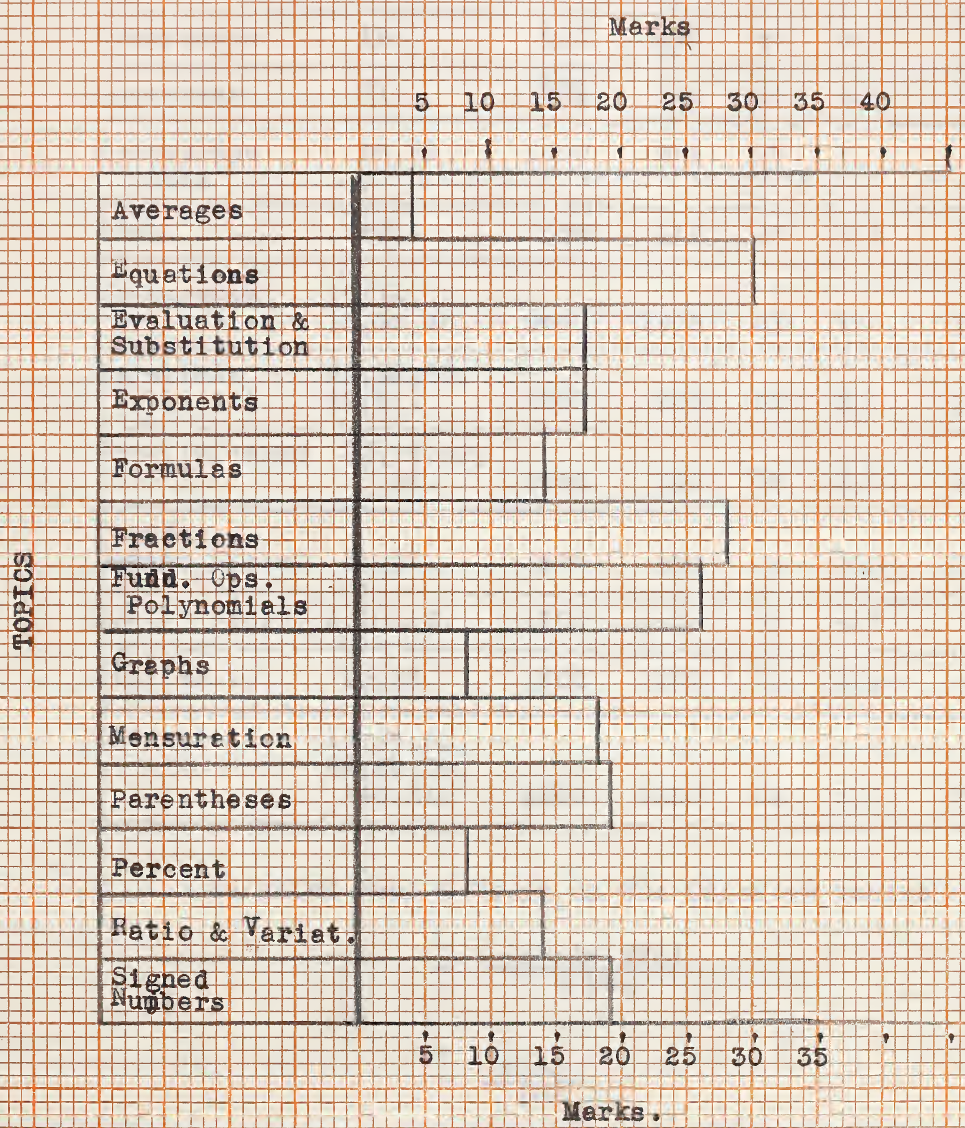


TABLE LXXXVIII

ACHIEVEMENT ON GEOMETRY TOPICS

Topic	Questions	TRSP	TRSO	%-A	MTS	TPS
Angles	2,10,12,14 18,21,23, 26,37.	5400	2540	47	8.5	18
Congruence	18.	600	242	40	0.8	2
Converses	21.	600	142	24	0.5	2
Hypotenuse Rule	33,39.	1200	294	25	1	4
Loci	29.	600	162	27	0.5	2
Measurement	10,14,15, 18,33,40.	3600	1470	41	5	12
Parallels	21,23,33, 40,51.	4200	915	22	3	14
Parallelogram	33,39,40, 52.	3600	655	18	2.2	12
Proofs- Harder probs.	51,52.	3600	138	4	0.5	12
Easier probs.	2,10,12.	1800	1416	79	4.7	6
Similarity	15,26.	1200	128	11	0.4	4
Triangles	15,18,21, 26,33,39, 40,51.	6000	1223	24	4.8	20

Weighted Mean % Score on the Geometry topics

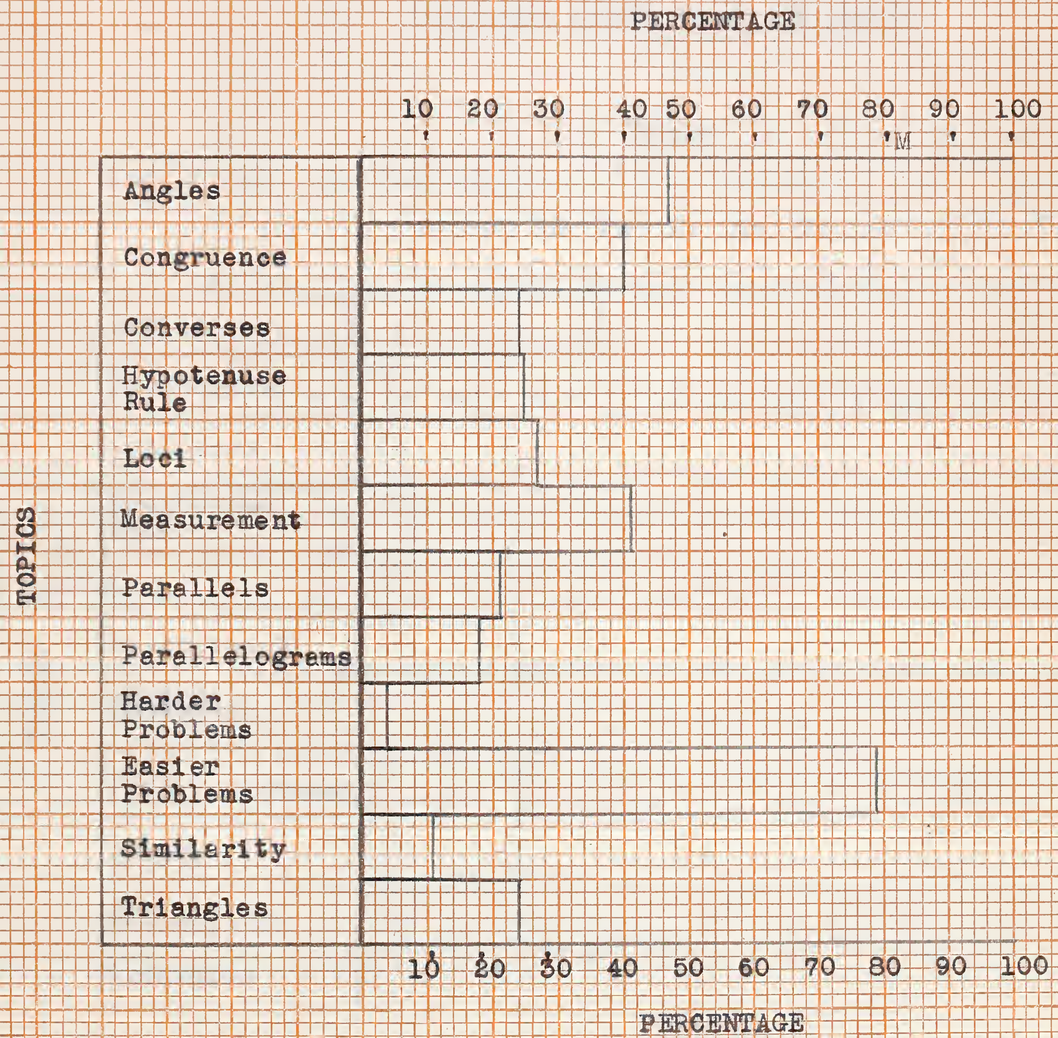
$$= \frac{\Sigma(\%\text{-A})x(\text{TPS})}{\Sigma\ \text{TPS}}$$

$$= \frac{3190}{108}$$

$$= 29.4$$

PERCENTAGE OF "TOTAL RAW SCORE POSSIBLE"
OBTAINED ON THE INDIVIDUAL TOPICS IN THE
GEOMETRY SECTION OF THE EXAMINATION.

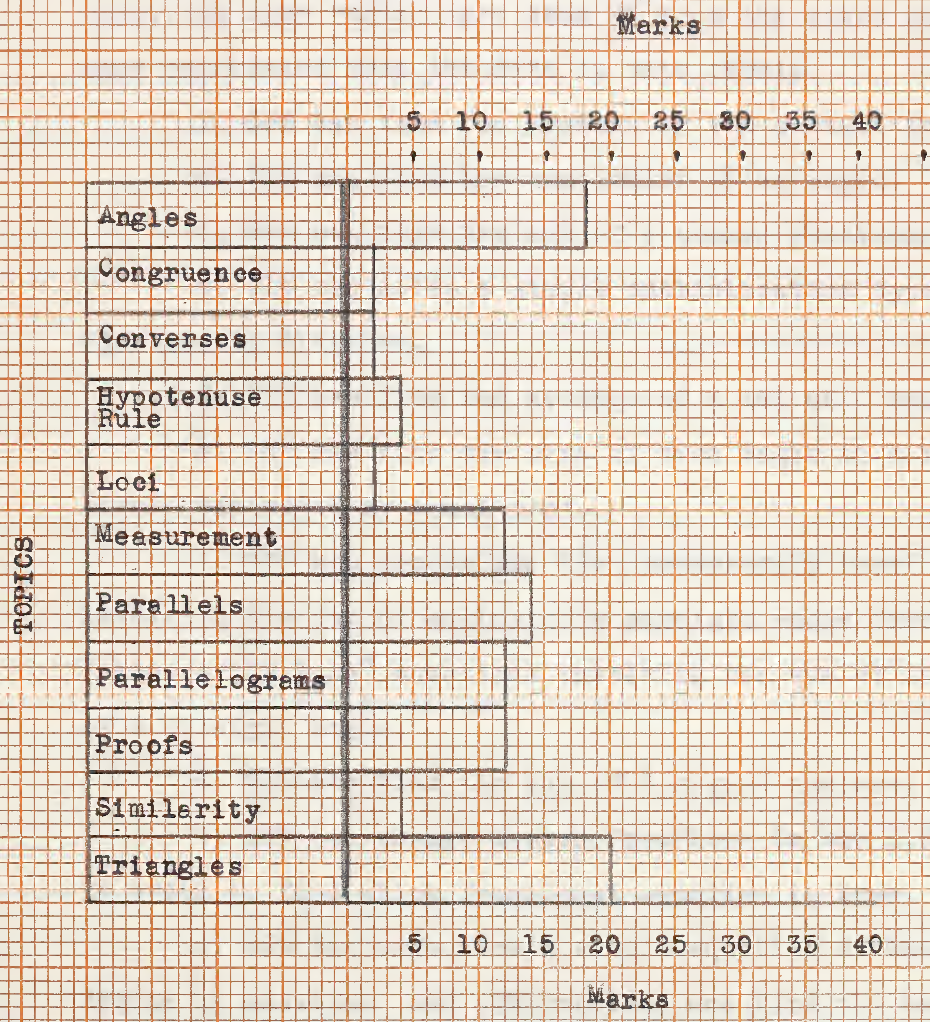
Figure 7.



The following graph shows the total raw score value for each of the main topics in the Geometry section of the examination paper.

Figure 8

TOTAL RAW SCORE VALUE PER TOPIC



CHAPTER IX

SUMMARY

In chapter I eight important questions were asked, answers to which adequately summarize the finding of this investigation.

A. What do the students at the lower limit of group B know?

The following percentages quoted are approximations only, but none varies more than $\frac{1}{2}\%$ from the exact percentages calculated from the data in this thesis.

1. 55% can find the product of two decimal fractions.

2. 71% can find the complement of an angle.

3. 87% can find the sum of 3 common fractions.

4. 61% can solve a simple multiplication type of equation by division.

5. 71% can find the average of a set of 5 numbers.

6. 88% can find the size of each angle in a semicircle subtended by equal arcs.

7. 77% can choose the "not necessarily true" equation involving the sum of angles of a straight angle from the "necessarily true" equations involving the sum of angles of a straight angle.

8. 81% can find the sum of 6 signed numbers.

9. 58% can change common fractions to the L.C.D. in order to find which fractional equation is true.

10. 65% can transform a simple equation of the first degree by addition and subtraction, and find the value of the unknown.

11. 51% understand the verbalization of the formula for finding the volume of a cylinder.

12. 57% can calculate the length of each side of a cube that has a volume of 512 cu. in.

13. 53% can accurately apply theorems 17,20,21, and 22, to select equalities applicable to a given figure.

14. 62% can find the product of a common fraction and a mixed fraction.

15. 75% can divide a common fraction by a common fraction.

It is interesting to note that almost every one of the 15 items mentioned above, involves "old material", or mathematics that the students learned in grades previous to grade IX. The writer feels quite sure that the mathematics understood by the great majority of these 300 students, would also be quite well understood by the "good average" pupil in grade VIII. Student achievement with new material, or mathematics of the ninth grade, is best seen in the list of 41 items that is to follow.

B. <u>What do the students at the lower limit of group B NOT know?</u>

1. 93% cannot verbalize the formula required for finding the area of the four walls of a room.

2. 81% cannot find the area of a circle of given diameter.

3. 57% cannot select from a group of fractions, a fraction or ratio equal to a given ratio.

4. 70% cannot calculate the area of a rectangle whose sides have all been increased 5 times.

5. 59% cannot find the value of an unknown quantity which will satisfy an equatinn of the first degree.

6. 67% cannot find the sum of the 5 interior angles of a pentagon.

7. 83% cannot find the length of one side of a triangle that is similar to another triangle.

8. 68% cannot expand $(-2a^3)^2$.

9. 60% cannot choose the pair of congruent triangles from a group of triangles of known dimenions.

10. 55% cannot find the product of a set of numbers composed of literal coefficients, numerical coefficients and positive and negative numbers.

11. 72% cannot change "inches per second" to "feet per minute".

12. 76% cannot select the "necessarily true" converse from a group of 5 propositions.

13. 63% cannot select the correct formula for finding the volume of a cylinder, nor evaluate the formula for the volume of a cylinder when the dimensions are given.

14. 54% find it impossible to read a given question correctly or else do not know that the sum of the angles of a straight angle is equal to 180 degrees.

15. 64% find it impossible to correctly evaluate a given formula with known quantities.

16. 65% cannot perform the fundamental operations on a group of common fractions to find the greatest value in the group.

17. 96% cannot apply the rules of similar triangles to a group of right angled triangles and correctly choose the corresponding parts which are not equal.

18% 69% cannot divide a product containing exponents by one of its known factors.

19. 73% do not know the locus of all points on a plane that are a given distance from 2 given points in that plane.

20. 88% cannot find a number that bears a certain ratio to a given number as 3 bears to 8.

21. 58% cannot properly divide a polynomial by a monomial.

22. 87% cannot apply either Theorem 17A or the Hypotenuse Rule to find the length of a side of a given figure.

23% 61% cannot find $\frac{1}{4}$% of 16.

24. 86% cannot find by what % a given ratio exceeds another given ratio.

25. 60% do not know that the area of a sector of a given circle varies directly as r^2A, where r is the radius, and A is the angle subtended by the arc of the sector.

26. 81% cannot apply the rule of transformation by addition and subtraction to an equation of the first degree.

27. 64% cannot find the area of a parallelogram by using the Hypotenuse Rule to first calculate the vertical height of the given parallelogram.

28. 58% cannot find the sum of a simple algebraic expression.

29. 68% cannot divide a simple decimal fraction by another decimal fraction.

30. 77% cannot subtract a polynomial from a polynomial.

31. 81% cannot divide a polynomial by a polynomial.

32. 60% cannot find the sum of two polynomials.

33. 75% cannot simplify an algebraic expression by removing brackets and collecting terms.

34. 63% cannot divided a binomial by a monomial.

35. 83% cannot apply the rule of transformation by addition and subtraction to a first degree equation.

36. 99% cannot apply assumption 2 and Theorem 21 to prove a simple geometric problem.

37. 100% of these students cannot prove that the bisectors of the angles of a parallelogram form a rectangle.

38. 98% of the students cannot read a graph sufficiently well to calculate oil production over a period of time.

39. 87% cannot calculate from a straight line and broken line graph, by what percentage Alberta's oil production in 1943 was below that of 1942.

40. 93% cannot interpret a graph sufficiently well to calculate oil production over two periods of time, then express these production figures as a ratio.

41. 99% cannot calculate Alberta's average annual oil production over a 13 year period, by interpreation of a broken line graph.

We noticed in part A of this summary that the greatest pupil success was with material that was actually a review of prefious years' mathematics. Almost all of the items appearing in part B are concerned with the relationships, concepts and routines of the grade IX mathematics course, and it is plainly seen that the vast majority of students know very little or have very little understanding of the mathematical fundamentals of the grade IX text.

C. Are definitions meaningful to the students?

In this thesis, definitions are synonymous with concepts or technical vocabulary. The Summary of Achievement in Concepts on page 143 shows that the percentage achievement on concepts was 36, which means that slightly over 1/3 of all the marks alloted for concepts were earned by the 300 students at the lower limit of group B. We can conclude from this that defintions, or concepts, are not meaningful to the great majority of these 300 students.

D. Do the students know their proofs and constructions?

This question may be answered with one word, "No". There were only two questions on the examination which tested the students' abilities to prove various theorems and propositions. One of these was much too difficult for all these students, but the other was very straightforward and fair. Yet, only one person out of 300 was able to get full marks for the easier proof, and no one got more than two out of six marks for the more difficutl question (question 52). Question 51, the easier proof,

was an application of assumption 2 and theorem 21, yet there were only 12 persons who got parital marks for proofs.

65 persons were able to get the correct construction for question 51, for which they were given one mark. Of these 65 persons, only 13 were able to proceed to the next step and apply theorem 21. Regarding question 52, no construction was required other than making the correct diagram, which only 40 persons were able to do correctly. From this evidence we may conclude that these students do not know proofs. It is unfortunate that the examination did not more thoroughly test for constructions, 8 of which appear in the text- none of which appears on the examination paper.

E. <u>Do the answers given show any evidence of logical thought, or does the work reflect only memorization?</u>

Evidence of frequent guessing, combined with the large number of very absurd responses show that critical thinking is not being done on the examination. Example, in question 7, 49 persons said that the fraction or ratio 5/8, which is less than 1, is actually equal to the ratio 8/5, which is greater than 1. In question 20, 146 students said that "20 ft. per minute" is equal to "240 inches per second". If these students had applied just the slightest bit of thought to the meaning of the problem, they would have seen that answer "b" was absurd. The height of absurdity was reached in question 53 where we see a range of answers for part b from 1/10% to 10 million percent. Although these are extreme examples being quoted, they nonetheless show that a great number of

students have chosen answers, or calculated results , which if given one moment of critical thought would have reflected the complete absurdity of the answer. This is certainly evidence that few if any of these students took the time to check their work, or make rough approximations as to the reasonableness of ther answers.

Memorization is reflected to a great extent in questions 51 and 52. Here may be found proofs for diagrams which don't even correspond with the given data of the question. One can only say that the wording of the problem of the figure itself, reminded many of a once known, but now practically forgotten proof, and they blithely attempted to write down what once had been memorized, but never completely understood.

F Does the examination paper test the students' powers to generalize or theorize?

Yes. The paper gives many chances to generalize. This is seen in such questions as 4,9,21, and 37 to mention only four specific questions. The results show that even though the students do have plenty of chances to generalize, their ability to do so is very limited.

Testing for powers of generalization is synonymous with testing for relationships. The Summary of Achievement in Relationships on page 141 shows that only 28.8% of the marks alloted to relationships were actually earned by the 300 students. This is surely evidence that the majority of these 300 students have not the ability to generalize, theorize, or see relationships.

G. Does the examination paper test the student's power to specialize and apply abstract theory to particuaar problems?

Specializing and applying abstract theory to problems has been classified as a routine in this thesis. The examination certainly does test the pupil's abilities to apply abstract theory- in fact 63% of the paper may be classified under routines, or application of theory, in contrast to 31% for relationships, and only 6% for concepts.

The Summary of Achievement in Routines on page 146 shows that only 35.9% of all the marks alloted for routines or applications, were earned. This is conclusive evidence that the great majority of the lower limit of group B students have not the ability to apply abstract theory to concrete situations. The fact that almost 8% of the marks allotted to routines and routine questions, were lost through no attempt being made to answer the problem is further evidence that quite a large number of students have not the slightest understanding of abstract theory or of its application in the solution of a problem.

H. Was the examination fair?

Fairness of the examination is best discussed under several headings, each of which contributes to an overall view of the fairness of the examination paper.

1. Text-book coverage

Generally speaking, the grade IX mathematics course as developed in the text 'Mathematics for Today' consists of 81 relationships, 125 routines and 303 concepts. Everyone of the relationships and routines required to answer the examination paper is found either in the authorized text-book, or else will be found in the text-books of earlier grades. All the concepts on the examination may be found in the text with the exception of the word "bears" which appears twice on the examination. In this respect, the examination paper content was very fair.

The examintion paper itself consisted of some 58 relationships, 106 routines, and 99 concepts. This corresponds favorably with the allocation of marks mentioned in part G.

One realizes that no examination can test for all the material covered in the course, but an examination that covers approximately 30% of the relationships, 85% of the routines, and 33% of the concepts is making a very good coverage of the year's work.

N.J. Lennes in an article 'The Function Concept in Elementary Algebra' as found in 'The National Council of Teachers of Mathematics- Seventh Year Book" makes the following statement: " It is often said that there are three essential elements in elementary algebra, namely: the use of letters to represent numbers (this includes the formula), the introduction of signed numbers, and the

use of the equation". 'Mathematics for Today' certainly covers these three essential elements.

In regard to the chapter 'Problems' in the text-book, it is the writer's opinion that this section was not sufficiently tested. This is one section of the course that can live for the students- they can see the practical application of mathematics to their everyday life. Yet there was not a single verbalized problem on the examination. Many students who cannot understand or solve an abstract mathematical formula, can nonetheless "tackle" what the text refers to as "miscellaneous problems or old fashioned problems" and get the correct answer. This, the writer feels is one weakness in the examination.

Another weakness that the writer feels exists in the examination is the absence of any constructions. Like problems in algebra, constructions in geometry are things that the student can see and do- they have a practical application to the individual student, as he can actually see the results of applying his geometry unfold before his very own eyes. It is my opinion that if the examinations have a factor of practical application, they will much better reveal what the students actually know, much more than an examintion based on the abstract.

Examination of the graphs on pages 155 and 158 shows that the marks as a whole were well distributed over the various algebra and geometry topics, but one wonders why so many marks were given for the two topics 'ratio' and 'percent'. There is very little discussion of the topic 'ratio and variation' in the text-book, and none at all in the grades 7 and 8 texts, yet, almost 10% of the

one of the exercise". "Mathematics for [illegible]" [illegible] [illegible] these essential elements.

In regard to the chapter 'Problems' in the textbook, it is the writer's opinion that this section was not sufficiently tested. This is one problem of the entire [illegible] that all live for the students - that they see the practical application of mathematics to their everyday life. Yet there was not a single correlated problem on the examination. Many students are simply unable to do problems on abstract mathematical formulas, and would [illegible] "Rather what the test refers to as "miscellaneous problems or old practical problems" and get the correct answer. Thus, the writer feels it was unwise in the examination.

Another weakness that the writer feels exists in the examination is the absence of any constructions. Also problems in algebra, constructions in geometry are things that the student can see use for. They are a practical application to the individual student, and he can actually see the results of applying the geometry which before his very own eyes. It is my opinion that if the examinations have a choice of practical applications, then all these [illegible] would suggest that the students certainly know much more than an examination based on the abstract.

Examination of the answers on pages 102 and 103 shows that the scores on a whole were well distributed over the various algebra and geometry topics, but some numbers [illegible] of each topic were given [illegible] very near to the [illegible] and [illegible]. There is very little discussion on the topic 'fractional equations' in the textbook, and none of all in the [illegible] [illegible] and [illegible] [illegible], yet, almost [illegible] of the

examination is devoted to questions involving ratio and variation.

Questions involving percent, comprise more than 8% of the examination. Even though the topic of percent occupies very little space in the text-book, its appearance on the examination may be justified in that it is a good test of the student's mastery of grade 7 and 8 mathematics fundamentals. If this examination paper is so designed to test mathematical achievement in grades 7,8, and 9 then the percent problems on the paper are justified.

2. Fairness of the test items

Appraisal of the 53 questions on the examination would have one conclude that the test items as a whole were quite fair- none could actually be called 'trick' questions, and the values assigned to each were generally speaking, proportional to the difficulty involved. One could perhaps say that such questions as 2,3,8,10, were overvalued in comparison to more difficult questions as 24,33, and 56, but it must be realized that it is quite an accepted procedure to begin an examination with a few of the easier problems, to encourage the poorer students if nothing else, and thus the criticisms that questions 2,3,8,10 are overvalued, are not fully justified.

The usual examination procedure of alternating the more difficult and the less difficult questions seems well carried out(see figures 2 and 3 in chapter III). On the whole, the easier problems appeared nearer to the beginning of sections A and B, and the more difficult ones were left nearer the end. Questions 26,30,33, which appear to be quite difficult for the majority of students

should have been placed nearer the end of the paper.

Some question could be cast as to the validity of 5 specific test items; 1,22,37,46, and 52. While it is evident that the examiner's intentions in question 1 were to test the student's understanding of decimal notation, it nonetheless sets a poor example in regard to significant figures. Speaking in strictly mathematical terms, the question asks for the product of two numbers, each of which has 4 significant figures. In no case could the product of these two numbers be significant to more than 4 figures, therefore the most exact answer that we could have would be 1219. One of Alberta's noted mathematics teachers said of question 1, " This question is abominable. It should be pointed out that ALL of the answers are wrong. The correct answer contains at most 4 sig. figures, and is therefore 1220. This is an exercise in significant figures, and the examination paper itself is teaching bad habits."

With reference to question 22, the students are to find the volume of a cylinder, and the unit of measure necessarily must be cubic measure, yet everyone of the suggested answers was expressed in "sq. inches". This is perhaps a printer's error, but even if it is, the 60 students choosing answer "b" were just as correct as the 112 students choosing answer "c", yet they received no credit for having the formulization for the volume of the cylinder expressed rightly.

The examiner, in composing the paper, made a slight error in the wording of question 37, with the

result that there are actually 3 correct answers instead of one. The question is worded, 'The area of a sector of a circle varies directly as..." The three correct answers are:

a.....the length of the radius "r"

b.....the size of the angle A subtended by the arc of the sector of the circle

c..... the size of the angle A and the length of the radius "r".

The question should have been worded, "The area of a sector of THE circle varies directly as..." and there would only have been one correct answer- answer "d".

Question 46 in section B was not a fair test of the grade IX student's ability to divide a polynomial by a binomial. Had this been a question on the grade X examination, the question would have been justified, but it is very unlikely that many grade IX mathematics teachers had shown the pupils how to divide a polynomial by a binomial when they are faced in the (third division) with (9a-2) ÷ (5a-3). The fact that only 19% of the 300 students got the correct answer, is evidence of the difficulty of this particular problem.

Question 50 was one of the best types of questions on the examination, and gave much evidence that insufficient stress is laid by teachers on the proper solution of first equations.

Question 52 was much too difficult for EVERY one of these 300 students. When not one person out of 300 receives full marks, when 39 receive one mark, and only one student receives 2 marks out of a possible total of 6 marks, there is justification for saying that this item was unfair.

It was perhaps put on the paper to test the "better student" but when so few geometry problems were asked on the examination, this should not have been one of them. It was very unfair to the great majority writing the paper, especially when it says on page 306 of the prescribed text,under Language Practice, "Express each of the following problems in correct English without reference to a lettered figure. NO PROOFS ARE REQUIRED. (capital letters are those of the writer). Question 52 with a slightly different wording, is identical in meaning. Such questions as 52 may be good supplementary problems for the clever students, but they have no place on an examination designed to test all the students, not just the superior. Had other more easy geometry problems involving required proofs been on the paper as well as question 52, then the examinations' committee would have been justified in setting this particular problem.

Question 53 was one of the most difficult and also one of the most poorly answered of all the questions. The wide variety of answers suggests a very inadequate preparation in the classroom for this particular type of graph. The major difficulty with question 53 was in the interpretation of the units as expressed on the graph. Some students took each small square as the unit of measure, some took the large square as the unit of measure, and the vast majority were bewildered and confused- not knowing what to take as the unit. Those that did choose the unit of measure correctly in most cases found it very difficult or impossible to change their units into percentage. Although this question cannot be called

unfair, it is certainly a type of graph with which few students have had any classroom experience, and it is certainly not the type of graph which one would expect after covering chapter IV in 'Mathematics for Today'.

3. Was the examination paper too long?

In the great majority of cases the students had plenty of time to read and answer the examination paper. The total number of marks lost on the 300 papers through "no attempts" comprises 4.2% of the total raw score possible, and in most cases it is very evident that these no attempts were due, not to insufficient time, but rather to an inability to answer the problem. Had the paper been too long a greater number of no attempts would have been in evidence for the latter questions of the examination paper. We can thus say that the paper was not too long.

4. Construction of the examination paper

The construction of the examination paper was such that it acted as a handicap to many students. The writer noticed quite a number of errors occurring in the copying of answers from the rough work to the space provided for the correct answer. In many cases, this was due to the rough work being performed on the back of one sheet, and the final answer being transferred to another page. If more space were given nearer to the questions, by spreading the blank pages at the end of the booklet throughout the examination paper, many of these errors would not have occurred.

The use of machine scoring type questions might also be criticized. It is very likely that many of

the incorrect answers in section A were due to human errors in marking the answer sheet. It is very easy to choose the correct answer from the booklet, and it is also just as easy to mark the wrong answer on the answer sheet. Children with eye defects would also be working under a handicap, especially with section A, where good eyesight is an absolute necessity.

The writer would also like to suggest that where fractions are printed on the examination paper, as in question 3, that bigger type be used. It is very easy to look at the fraction 1/3 as printed on the booklet, and read it as 1/8. Similarly in question 7, a student with poor eyesight might readily mistake the ratio 5/8 for 5/3. Larger sized type is used on the 'suggested answer side' of the examination booklet, and there is no reason why such type could not be used on the left hand side of the booklet as well. It would materially help those suffering from poor vision.

TABLE LXXXIX

SUMMARY OF SCORES

Q	FM	Z	N	NA	V	NV	FMV
1	164	136	298	2	2	596	328
2	212	88	299	1	2	598	424
3	261	39	298	2	2	596	522
4	22	278	300	0	2	600	44
5	184	116	290	10	2	580	368
6	56	244	289	11	2	578	112
7	129	171	293	7	2	586	258
8	233	67	299	1	2	598	466
9	91	209	299	1	2	598	182
10	264	36	298	2	2	596	528
11	122	178	279	21	2	558	244
12	232	68	299	1	2	598	464
13	242	58	296	4	2	592	484
14	99	201	300	0	2	600	198
15	51	249	299	1	2	598	102
16	95	205	300	0	2	600	190
17	173	127	290	10	2	580	346
18	121	179	299	1	2	598	242
19	136	164	289	11	2	578	272
20	84	216	293	7	2	586	168
21	71	229	298	2	2	596	142
22	112	188	281	19	2	562	224
23	138	162	300	0	2	600	276
24	107	193	275	25	2	550	214
25	105	195	295	5	2	590	210
26	13	287	295	5	2	590	26
27	92	208	296	4	2	592	184
28	195	105	283	17	2	566	390
29	81	219	292	8	2	584	162
30	36	264	278	22	2	556	72
31	125	175	280	20	2	560	250
32	154	146	296	4	2	592	308
33	40	260	288	12	2	576	80
34	171	129	279	21	2	558	342
35	117	183	296	4	2	592	234
36	41	259	279	21	2	558	82
37	120	180	283	17	2	566	240
38	58	242	279	21	2	558	116
39	107	193	277	23	2	554	214
40	160	140	291	9	2	582	254
41	127	173	294	6	1	294	9962
42	96	204	290	10	1	290	
43	186	114	295	5	1	295	
44	226	74	293	7	1	293	
45	67	232	293	7	2	586	
46	44	256	253	48	2	506	
47	83	180	240	60	3	720	
48	76	224	277	23	2	554	
49	110	190	267	33	2	534	
50	55	230	219	81	4	876	
51	1	235	262	38	6	1572	
52	0	260	229	71	6	1374	
53	0	234	288	12	8	2304	
						33494	

KEY for table LXXXIX

Q: Question

FM: Frequency of full marks

Z: Frequency of zero marks

N: Number of students attempting the particular question

NA: Number of students not attempting question

V: Value of question

NV: Number of marks attempted for the particular question (the product of N and V)

FMV:-Total number of marks earned for the particular question by all the students. (applicable only to questions 1-40 inclusive, where no partial marks were given.)

The most informative column in table LXXXIX is "NV", which represent the total number of marks attempted for each question. The sum of the NV column would therefore represent the total number of marks attempted for the whole paper. This sum total is 33,494 marks. If this sum total then be divided by the total number of students (300) we would then have the "average number of marks" attempted per paper.

$$\text{Average No. of Marks attempted per paper} = \frac{33494}{300}$$

$$= 111.65 \text{ marks.}$$

It will be remembered that the total value of the paper is 119 marks. From this we can calculate the percentage of the paper that was attempted.

$$\text{Percentage of Paper attempted} = \frac{111.65 \times 100}{119}$$

$$= 93.8$$

It is very significant that 93.8% of the paper was attempted, but two other important questions arise. What was the average number of marks per paper actually earned? What was the percentage of the paper actually earned?

The total number of marks actually earned for each of the first 40 questions is listed under the heading FMV in the Summary of Results on page 177. There were no partial marks given for these 40 questions- the scores per question were either zero or two marks. Thus by multiplying column "FM" and column "V" it was possible to find the total number of marks actually earned for each of the 40 multiple choice items. The Summary of Scores shows that the total number of marks actually earned for the first 40 questions was 9962 marks. Table XC on the following page gives a summary of the marks actually earned for questions 41-53 inclusive. By adding the total of "FMV" in table LXXXIX to the "Total raw score" in table XC it will be possible to find the total number of marks earned on the full paper.

SUMMARY OF SCORES ON QUESTIONS 41-53 INCLUSIVE

The total of the column "FMV" in table LXXXIX on page 177 represents all the marks made by the 300 students on the first 40 questions. The table below is a summary of all the marks earned by all the students on question 41-53 inclusive, and includes all partial marks as well as perfect scores.

TABLE XC

RAW SCORES OBTAINED (Q 41-53 inclus.)

Q	FS	SV	TRS
41	173	0	0
41	127	1	127
42	204	0	0
42	96	1	96
43	114	0	0
43	186	1	186
44	74	0	0
44	226	1	226
45	232	0	0
45	1	1	1
45	67	2	134
46a	242	0	0
46a	58	1	58
46b	256	0	0
46b	44	1	44
47	180	0	0
47	37	1	37
47	0	2	0
47	83	3	249
48	224	0	0
48	0	1	0
48	76	2	152
49	190	0	0
49	0	1	0
49	110	2	220
50	230	0	0
50	11	1	11
50	4	2	8
50	0	3	0
50	55	4	220
51	235	0	0
51	52	1	52
51	5	2	10
51	2	3	6
51	2	4	8
51	3	5	15
51	1	6	6

KEY

Q: question

FS: frequency of score

SV: score value

TRS: total raw score obtained

TABLE XC (cont'd.)

RAW SCORES OBTAINED (questions 41-53)

Q	FS	SV	TRS
52	260	0	0
52	39	1	39
52	1	2	2
52	0	3	0
52	0	4	0
52	0	5	0
52	0	6	0
53a	293	0	0
53a	0	1	00
53a	7	2	14
53b	260	0	0
53b	0	1	0
53b	40	2	8
53c	279	0	0
53c	0	1	0
53c	21	2	42
53d	297	0	0
53d	0	1	0
53d	3	2	6
			2049

Total marks earned on first 40 questions is 9962 marks. (from FMV in table LXXXIX). The total raw score, or actual number of marks earned on the questions 41-53 inclusive is 2049 marks (from the summation of TRS in table XC). Therefore the total number of marks actually earned on the whole paper by all students in this investigation is 9962 + 2049, or 12,011 marks.

With the raw score value of the whole paper as 119, and 300 students writing the examination (in this investigation) it was potentially possible for 35,700 marks to be earned.

$$\text{Average no. of marks earned per paper} = \frac{12011}{300}$$

$$= 40.04 \text{ marks.}$$

Percentage of marks earned by the 300 students

$$= \frac{40.04}{119} \times 100$$

$$= 33.6$$

CHAPTER X

CONCLUSIONS

The standard of achievement for the 300 students at the lower limit of group B in the grade IX mathematics examination in June 1948, may be summarized with the following conclusions:

1. The great majority of questions that were quite well answered were those involving routines and relationships learned previously in grades 7 and 8.

2. The most poorly answered questions were those involving relationships, routines, and concepts learned in grade IX.

3. Generally speaking, definitions and concepts are not meaningful as they should be, when we consider that these students all hold B standing in grade IX mathematics.

4. Over 95% of these students do not know proofs, theorems, and assumptions sufficiently well to get even half marks for those questions requiring an understanding of the above.

5. There is very little evidence of logical thought or reasoning entering into the solution of most problems, and very few if any of the students concerned themselves with checking answers or making rough approximations of the correct solution.

6. The students are not able to generalize, theorize or see relationships. There is a complete lacking in functional thinking. To Professor Felix Klein this would be appalling, as he states in the National Council of Teachers of Mathematics- Seventh Year-book, "Functional thinking is the very soul of mathematics.

7. The vast majority have little ability or power to apply abstract theory to a problem, or use the techniques of the grade IX mathematics course in solving given problems.

8. The average number of marks attempted per paper was 112 out of a possible total of 119 marks.

9. The percentage of marks attempted per paper was 94.

10. The average number of marks earned per paper was 40 marks.

11. The percentage of marks earned was 34.

12. % achievement on relationships was 29.2

13. % of marks lost through not attempting problems involving relationships was 6.7

14. % achievement on concepts was 35.9

15 % of marks lost through not attempting problems involving concepts was 2.9

16. % achievement on routines was 35.9

17. % of marks lost through not attempting problems involving routines was 7.7

18. % of marks lost through mechanical errors in addition and subtraction was 9.9

19. % of marks lost through mechanical errors in multiplication and division was 8.4

20. Total % of marks lost through all types of mechanical errors was 18.3

The following are the special weaknesses as revealed by this investigation:

1. The vast majority of these 300 students have

not the ability to recognize and understand dependence and relationships, which are the most familiar aspects of the function concept. A.E. Breslich says in the Seventh Year-book, "recognition of the dependence of one variable quantity on another related variable is..... one of the most important aspects of functional thinking."

2. Many students do not understand the use of the L.C.D. in regards to testing the equality of fractions and ratios.

3. There is much weakness in the fundamental operations of signed numbers.

4. Very few students have been taught to check their answers, either by approximation, or by inspection.

5. Few of these students can understand or interpret graphs and graphical units and concepts.

6. Few students in this group understand the meaning of "percent" and thus find it impossible to solve simple percentage problems or more complicated graphical problems related to percent.

7. The concept of similarity and similar triangles, and their use in comparing figures by ratio and proportion is not well understood.

8. The majority of these students are very weak in the four fundamental arithmetic operations, as seen in this investigation- where mechanical errors accounted for over 18% of the lost marks.

The conclusions reached in this investigation correlated highly with the conclusions of Mr. Hooper, Mr. H. Miller, Miss V. Miller and Dr. Lazerte. The

pupils as a group have a weak background in mathematics-they have not the ability to think clearly or analyze problems, and the frequency of guessing, combined with the large number of very absurd responses show that critical thinking is not being done on the examination.

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HIGH SCHOOL ENTRANCE EXAMINATION BOARD

DEPARTMENTAL EXAMINATIONS, 1948

GRADE IX

MATHEMATICS

Time—$2\frac{1}{2}$ hours.

Note—The total time allowed for this paper is TWO AND ONE - HALF HOURS. Distribute this time to the best advantage, reading the paper, answering the questions and reviewing the work already done. Do not spend too much time on any one question.

SECTION A

Section A of the paper contains questions 1 to 40 inclusive. Answers to these questions are to be recorded on the separate ANSWER SHEET which is loosely inserted in the examination booklet. Each question has five suggested answers, only one of which is correct. Select the correct answer in each case and record your choice on the separate answer sheet provided, as shown in the sample below:

SAMPLE:

	Answers			Answer Page				
				a	*b*	*c*	*d*	*e*
The product of 7 and 8 is	*a.* 78	*b.* 48						
	c. 56	*d.* 72	*e.* 15	⁞	⁞	▮	⁞	⁞

Be sure to mark your answer distinctly, using a soft lead pencil and making a heavy black mark. If you wish to change your answer, erase your first mark completely.

SECTION B

Section B of the paper contains questions 41 to 53 inclusive. Complete solutions for each of these should be shown in the spaces provided in the booklet.

Note—Space is provided in the booklet for rough work for all 53 questions.

Do not write your name on either the booklet or the separate answer sheet.

Fold both booklet and answer sheet once, and place them separately (*i.e. not folded together*) in the same envelope.

CANDIDATE'S NUMBER

(For the use of sub-examiners only)

Values

SECTION A

2 marks
for each

1. The product of 98.05 and 12.44 is

1. *a.* 12.19742 *b.* 121.9742
c. 1219.742 *d.* 12197.42
c. 12197.420

2. The complement of an angle of 40 degrees is an angle of .... degrees.

2. *a.* 140° *b.* 90°
c. 60° *d.* 320° *e.* 50°

3. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} =$

3. *a.* $\frac{3}{10}$ *b.* $1\frac{1}{30}$
c. $1\frac{4}{10}$ *d.* $\frac{1}{30}$ *e.* $\frac{1}{10}$

4. Which of the following statements is true?

4. *a.* The area of the walls of a room is the length of the room multiplied by the width of the room.
b. The area of the four walls is the product of the perimeter of the room and its height.
c. The area of the four walls is the product of the length, the width and the height.
d. The area of the four walls is four times the area of any one wall.
c. The area of the four walls of a room is the sum of its length and width multiplied by its height.

5. When 16 is multiplied by N the product is 96. What is the product when 16 is multiplied by $(N + 2)$?

5. *a.* 112 *b.* 128
c. 144 *d.* 32 *c.* 160

6. A circle has a diameter of 15 inches. Its area (sq. in.) is

6. *a.* 7.5π *b.* 15π
c. $(7.5)^2\pi$ *d.* $(15)^2\pi$ *e.* $(7.5\pi)^2$

7. Which of the following ratios is equal to the ratio $\frac{5}{8}$?

7. *a.* $\frac{3}{8}$ *b.* $\frac{4}{7}$ *c.* $\frac{1}{3}$
d. $\frac{1.5}{2.4}$ *c.* $\frac{8}{5}$

8. The average of 16 ft., 18 ft., 24 ft., 26 ft. and 30 ft. is

8. *a.* 24 ft. *b.* 22.8 ft.
c. 114 ft. *d.* 57 ft. *e.* 20 ft.

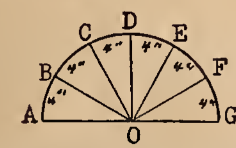
FOR ROUGH WORK

Values

9. The area of a rectangle is x sq. in. If each side were made 5 times as long, the area would be

9. *a.* $5x$ sq. in. *b.* $10x$ sq. in.
c. $15x$ sq. in. *d.* $20x$ sq. in.
e. $25x$ sq. in.

10.



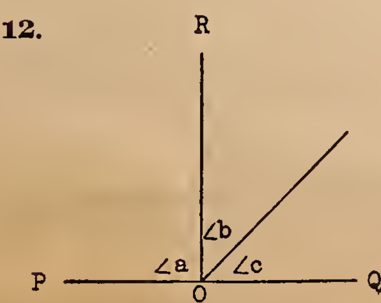
ABCDEFG is a semicircle. How large is each angle at 0?

10. *a.* 10° *b.* 15°
c. 30° *d.* 45° *e.* 60°

11. Which value of '*a*' satisfies the equation:
$2(a - 5) = 3(1 - a)$?

11. *a.* 5 *b.* 2.6
c. $\frac{5}{13}$ *d.* $2\frac{2}{3}$ *e.* $4\frac{1}{3}$

12.



POQ is a straight line and *RO* ⊥ *PQ*. Which of the following equations is not necessarily true?

12. *a.* $\angle a + \angle b + \angle c = 180°$
b. $\angle a = \angle b + \angle c$
c. $\angle b + \angle c = 90°$
d. $\angle b = \angle c$
e. $\angle a = 90°$

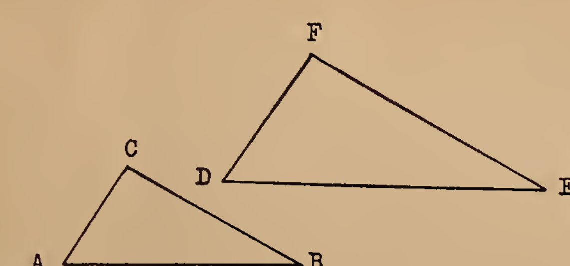
13. The sum of (+9), (–15), (–21), (–6), (+16) and (+4) is

13. *a.* –73 *b.* –23
c. –13 *d.* +13 *e.* +73

14. The five interior angles of a pentagon are together equal to

14. *a.* 360° *b.* 450°
c. 500° *d.* 540° *e.* 720°

15.



ABC and *DEF* are two similar triangles. $AC = 5''$, $CB = 8''$, $DF = 7''$. The length of *FE* is

15. *a.* 10″ *b.* $10\frac{1}{5}$″
c. 11″ *d.* $11\frac{1}{5}$″ *c.* 12″

16. $(-2a^3)^2$ is equal to

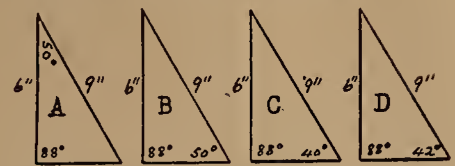
16. *a.* $-2a^6$ *b.* $-4a^5$
c. $+4a^6$ *d.* $+4a^5$ *e.* $+2a^6$

17. Which of the following equations is true?

17. *a.* $\frac{1}{2} = \frac{3}{4}$ *b.* $\frac{2\frac{1}{2}}{3} = \frac{4}{5}$
c. $\frac{5}{6} = \frac{12}{10}$ *d.* $\frac{3}{4} = \frac{2\frac{1}{2}}{3\frac{1}{2}}$
c. $\frac{5}{6} = \frac{2\frac{1}{2}}{3}$

Values

18. Four triangles (A), (B), (C) and (D) have dimensions as shown in the figures below.



Which pair of triangles is congruent?

18. *a.* A and B *b.* A and C *c.* B and C *d.* B and D *e.* A and D

19. The product of ($-x$), ($+2y$), ($-3y$), ($-4y$), ($-5x$) and ($+6y$) is

19. *a.* $-720xy$ *b.* $+720xy$ *c.* $-720x^2y^4$ *d.* $+720x^2y^4$ *e.* $+720x^3y^3$

20. How many "inches per second" are equivalent to "20 ft. per minute"?

20. *a.* 1200 *b.* 240 *c.* $\frac{1}{3}$ *d.* 4 *e.* $\frac{3}{5}$

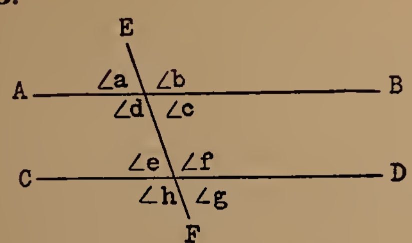
21. For which one of the following propositions is the converse necessarily true?

21. *a.* If I am sick, I do not go to school.
b. Triangles on the same base and between the same parallels are equal.
c. Triangles which have corresponding sides equal have corresponding angles equal.
d. The angles at the base of an isosceles triangle are equal.
e. Angles which are vertically opposite are equal.

22. The diameter of the base of a cylinder is 6 inches and its height is 20 inches. The volume is

22. *a.* πrh or $377\frac{1}{7}$ sq. in.
b. πr^2h or $2262\frac{6}{7}$ sq. in
c. πr^2h or $565\frac{5}{7}$ sq. in.
d. πrh^2 or $7542\frac{6}{7}$ sq. in.
e. πrh^2 or $3771\frac{3}{7}$ sq. in

23.

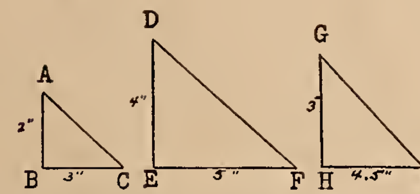


The two parallel lines AB,

23. *a.* $\angle d + \angle h = 180°$
b. $\angle d + \angle g = 180°$

Values

26.



ABC, DEF and GHK are three right-angled triangles. The lengths of two sides of each triangle are as indicated in the figures. Which of the equations is not true? 26

27. One factor of $15a^8b^6$ is $3a^2b^3$. The other factor is 27

28. If $2(4n+1) = 3(5n-2)$, which of the statements is true? 28

29. A and B are two points 7 inches apart on a plane. The locus of all points on that plane that are 4 inches from A and 6 inches from B is 29

30. What number, whole or fractional, bears the same ratio to $1\frac{1}{3}$ that 3 bears to 8? 30

31. When $ab^4 - 3a^3b^3 - 5a^4b^2$ is divided by ($-ab^2$), the quotient is 31

32. To find the volume of a cylinder 32

Values

34. How long is each side of a cube that has a volume of 512 cu. in.?

34. *a.* 6″ *b.* 7″ *c.* 8″ *d.* 9″ *e.* 10″

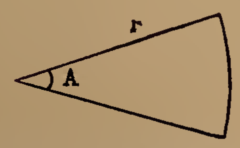
35. $\frac{1}{4}$% of 16 is

35. *a.* .04 *b.* .4 *c.* 4 *d.* 40 *e.* 64

36. By what per cent does the ratio $\frac{5}{4}$ exceed the ratio $\frac{6}{5}$?

36. *a.* $4\frac{1}{6}$ *b.* 5 *c.* 10 *d.* $16\frac{2}{3}$ *e.* 25

37.



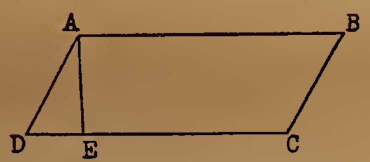
The area of a sector of a circle varies directly as

37. *a.* A *b.* r^2 *c.* rA *d.* r^2A *e.* r^2A^2

38. If $\dfrac{y-1}{3} = \dfrac{1}{4}(y+4)$, which of the equations is true?

38.
a. $3(y+4) = \frac{1}{4}(y-1)$
b. $4(y-1) = \frac{1}{3}(y+4)$
c. $12(y-1) = 3(y+4)$
d. $\frac{1}{4}(y-1) = \frac{1}{3}(y+4)$
e. $4(y-1) = 3(y+4)$

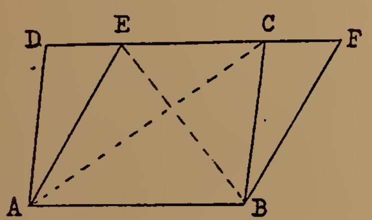
39.



ABCD is a parallelogram. $AE \perp DC$. If $AB = 4\frac{1}{2}''$, $AD = 2\frac{1}{2}''$ and $DE = 1\frac{1}{2}''$, then area of the parallelogram *ABCD* is

39. *a.* 7 sq. in. *b.* $7\frac{1}{2}$ sq. in. *c.* 8 sq. in *d.* $8\frac{1}{2}$ sq. in. *e.* 9 sq. in.

40.



In the given figure $DF \parallel AB$ and $AB = DC = EF$. Which of the statements is not true?

40.
a. Area $\|^m$ *ABCD* = Area $\|^m$ *ABFE*
b. Area $\triangle ABC$ = Area $\triangle AEB$
c. Area $\|^m$ *ABCD* = 2 × area $\triangle ADC$
d. Area $\triangle BCF = \frac{1}{2}BC \times CF$
e. $AE = BF$

FOR ROUGH WORK

SECTION B

Values

Simplify:

Write answers in the rectangles below.

1 **41.** $\frac{1}{5}x + \frac{5}{6}x - \frac{1}{2}x$ — **41.**

1 **42.** $1.4 \div 4.2$ — **42.**

1 **43.** $\frac{5}{6} \times 3\frac{1}{3}$ — **43.**

1 **44.** $\frac{4}{5} \div \frac{9}{10}$ — **44.**

2 **45.** From $6a^3 - 5a^2 + 3a - 14$ subtract $2a^3 - 4a + 7a^2 - 17$ — **45.**

2 **46.** Divide $20a^3 - 2 - 27a^2 + 18a$ by $(5a - 3)$. — **46.** Quotient is

Remainder is

3 **47.** If $P = 2n^2 + 7n - 15$, $Q = 10n^2 - 29n + 21$ and $R = 2n - 3$, find $(P + Q) \div R$. — **47.**

2 **48.** Simplify by removing brackets and collecting terms: $5a - 3(a-4) + 2(1-a) - (2a-5)$ — **48.**

2 **49.** Simplify: $\dfrac{12m^6n^5 - 30m^8n^7}{6m^2n}$ — **49.**

4 **50.** If $\frac{7}{12}K - \frac{1}{4}K = 2K - 1\frac{2}{3}$ find the value of K. — **50.**

FOR ROUGH WORK

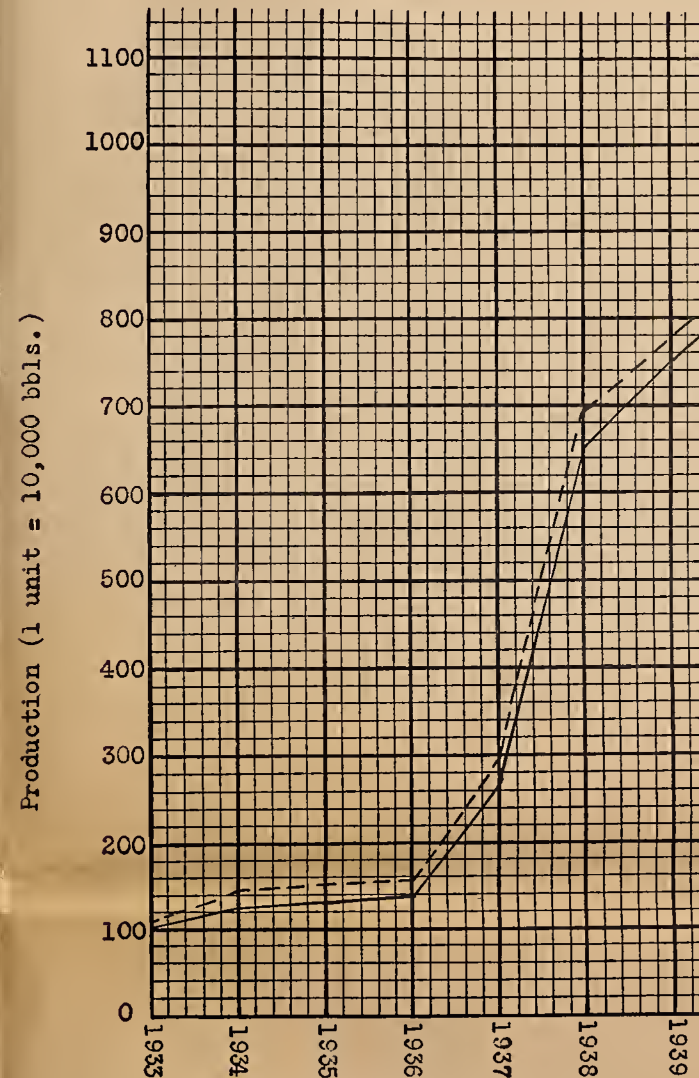
Values

6 **51.** A line parallel to the base BC of a triangle ABC cuts the sides AB and AC at D and E. Prove that ABE and ACD are equal in area.

FOR ROUGH WORK

6 **52.** Prove that the bisectors of the angles of a parallelogram form a rectangle.

FOR ROUGH WORK



Values

Years

Production of crude petroleum in A
10,000 bbls.), 1933-45.

2 **53.** (*a*) By what percentage (approx
did Canada's production in 1938

CANDIDATE'S NUMBER

(For the use of the Department only)

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